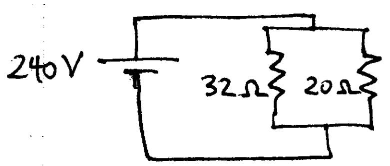
PHYSICS 220: Physical Electronics — Problem Set #1 SOLUTIONS

University Physics (11th ed.) Ch. 26 : 1, 2, 5, 6, 8, 9, 10, 19, 20, 21
Horowitz and Hill, Ch. 1 : 9

UP 26.1

The circuit described in this problem looks like this...



a) What is the resistance of the parallel combination?

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or...} \quad R_{eff} = \frac{R_1 R_2}{R_1 + R_2}$$

In this case $R_{eff} = \dfrac{32\Omega \cdot 20\Omega}{52\Omega} = \boxed{12.3\,\Omega}$

less than either R_1 or R_2.

b) What is the total current through the parallel combination?

$$I = \frac{V}{R_{eff}} = \frac{240\text{V}}{12.3\Omega} = \boxed{19.5\text{ A}}$$

c) What is the current through each resistor?

In parallel, the resistors have the same voltage across them. So $I_1 = \dfrac{V}{R_1}$ and $I_2 = \dfrac{V}{R_2}$

The current through the 32Ω resistor is.. $I_1 = \dfrac{240\text{V}}{32\Omega} = \boxed{7.5\text{ A}}$

The current through the 20Ω resistor is.. $I_2 = \dfrac{240\text{V}}{20\Omega} = \boxed{12.0\text{A}}$

Note that $I_1 + I_2 = 7.5\text{A} + 12.0\text{A} = 19.5\text{ A}$... as found in part b) and in agreement with Kirchoff's Junction Rule.

UP 26.2 Prove that when two resistors are connected in parallel, the equivalent resistance of the combination is always smaller than that of either resistor.

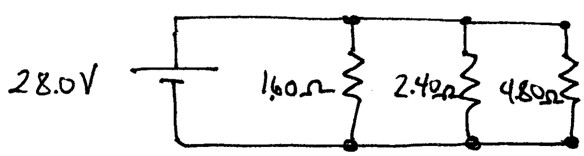
Parallel combination of two resistors: $R_{eff} = \dfrac{R_1 R_2}{R_1 + R_2}$ or... $\dfrac{1}{R_{eff}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Since resistances are positive quantities... $\dfrac{1}{R}$ is also a positive quantity...

Clearly $\dfrac{1}{R_{eff}} > \dfrac{1}{R_1}$ and $\dfrac{1}{R_{eff}} > \dfrac{1}{R_2}$

It follows that $R_{eff} < R_1$ and $R_{eff} < R_2$ QED

UP 26.5 The circuit described in the problem looks like this...



a) Find the equivalent resistance of the combination...

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = 0.625\,\Omega^{-1} + 0.417\,\Omega^{-1} + 0.208\,\Omega^{-1}$$

$$\frac{1}{R_{eff}} = 1.25\,\Omega^{-1} \qquad \boxed{R_{eff} = 0.800\,\Omega}$$

b) Find the current in each resistor...

$$I_1 = \frac{V}{R_1} = \frac{28.0\,V}{1.60\,\Omega} = \boxed{17.5\,A} \qquad I_2 = \frac{V}{R_2} = \frac{28.0\,V}{2.40\,\Omega} = \boxed{11.67\,A}$$

$$I_3 = \frac{V}{R_3} = \frac{28.0\,V}{4.80\,\Omega} = \boxed{5.83\,A}$$

c) Find The total current is... $I_{TOT} = I_1 + I_2 + I_3 = \boxed{35.0\,A}$ or.. $I_{TOT} = \dfrac{V}{R_{eff}} = \dfrac{28.0V}{0.800\,\Omega} = \underline{\underline{35.0\,A}}$

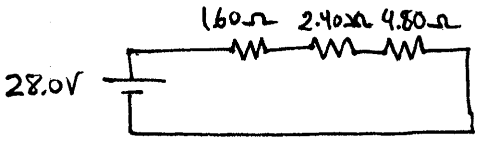
d) Find the voltage across each resistor... Since they are connected in parallel,... they have the same voltage.. $\boxed{V_1 = V_2 = V_3 = 28.0\,V}$

e) Find the power dissipated in each resistor...

$$P_1 = \frac{V^2}{R_1} = \boxed{490\,W} \qquad P_2 = \frac{V^2}{R_2} = \boxed{327\,W} \qquad P_3 = \frac{V^2}{R_3} = \boxed{163\,W}$$

f) The smallest resistor dissipates the most power, because they are in parallel. The voltage is the same, but the smallest resistor has the largest current. $P = IV$.

UP 26.6 Repeat the previous problem with the three resistors connected in series.



a) Find the equivalent resistance

$$R_{eff} = R_1 + R_2 + R_3 = 1.60\Omega + 2.40\Omega + 4.80\Omega = \boxed{8.80\Omega}$$

b) Find the current in each resistor. Since they are connected in series, they have the same current...

$$I = \frac{V}{R_{eff}} = \frac{28.0V}{8.80\Omega} = \boxed{3.18A} = I_1 = I_2 = I_3$$

c) Find the total current through the battery... $I_{tot} = I_1 = I_2 = I_3 = \boxed{3.18A}$

d) Find the voltage across each resistor.

$$V_1 = IR_1 = 3.18A \cdot 1.60\Omega = \boxed{5.10V}$$
$$V_2 = IR_2 = 3.18A \cdot 2.40\Omega = \boxed{7.63V}$$
$$V_3 = IR_3 = 3.18A \cdot 4.80\Omega = \boxed{15.26V}$$

Note that $V_{battery} = V_1 + V_2 + V_3 = 5.10V + 7.36V + 15.26V \cong \underline{28\ V}$

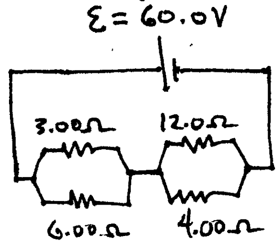
e) Find the power dissipated in each resistor.

$$P_1 = I^2R_1 = (3.182A)^2 \cdot 1.60\Omega = \boxed{16.2W}$$
$$P_2 = I^2R_2 = (3.182A)^2 \cdot 2.40\Omega = \boxed{24.3W}$$
$$P_3 = I^2R_3 = (3.182A)^2 \cdot 4.80\Omega = \boxed{48.6W}$$

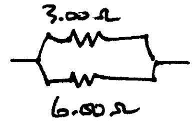
f) The largest resistor dissipates the most power because, when connected in series, the current in each resistor is the same, and $P = I^2R$.

UP26.8

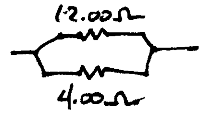
Compute the equivalent resistance of the network:



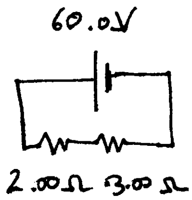
First combine the pairs of parallel combinations...



$= \quad R = \dfrac{3.00\,\Omega \cdot 6.00\,\Omega}{9.00\,\Omega} = \underline{\underline{2.00\,\Omega}}$

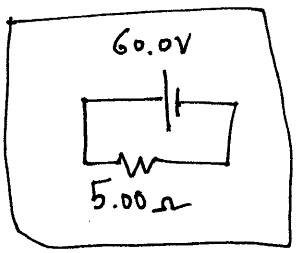


$= \quad R = \dfrac{4.00\,\Omega \cdot 12.0\,\Omega}{16.00\,\Omega} = \underline{\underline{3.00\,\Omega}}$



Then combine the series combinations...

$2.00\,\Omega \;\; 3.00\,\Omega \; = \; R = 2.00\,\Omega + 3.00\,\Omega = \underline{\underline{5.00\,\Omega}}$

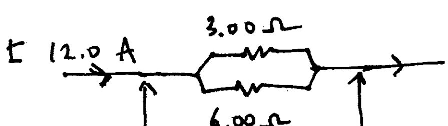


This is the equivalent resistance network.

Find the current in each resistor:

The total current is... $I_{Tot} = \dfrac{60.0\,V}{5.00\,\Omega} = \boxed{12.0\,A}$

Can now calculate the voltage across each parallel combination above....



$V = I \cdot R_{eff}$

$= 12.0\,A \cdot 2.00\,\Omega$

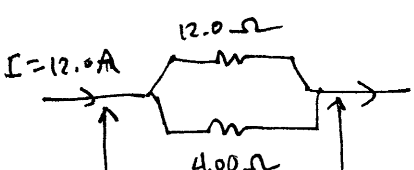
$= \underline{\underline{24.0\,V}}$

So, the current through the $3\,\Omega$ resistor is...

$\cancel{R} \quad I = \dfrac{V}{R} = \dfrac{24.0\,V}{3.00\,\Omega} = \boxed{8.00\,A}$

And the current through the $6\,\Omega$ resistor is...

$I = \dfrac{V}{R} = \dfrac{24.0\,V}{6.00\,\Omega} = \boxed{4.00\,A}$



$V = I R_{eff}$

$= 12.0\,A \cdot 3.00\,\Omega$

$= \underline{\underline{36.0\,V}}$

So, the current through the $12.0\,\Omega$ resistor is

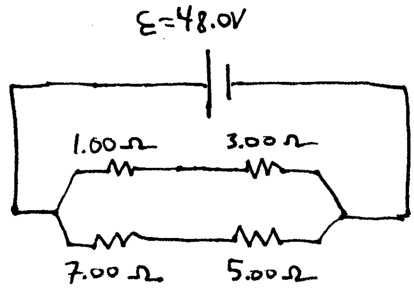
$I = \dfrac{V}{R} = \dfrac{36.0\,V}{12.0\,\Omega} = \boxed{3.00\,A}$

and the current through the $4.00\,\Omega$ resistor is..

$I = \dfrac{V}{R} = \dfrac{36.0\,V}{4.00\,\Omega} = \boxed{9.00\,A}$

UP 26.9

Compute the equivalent resistance of the network:



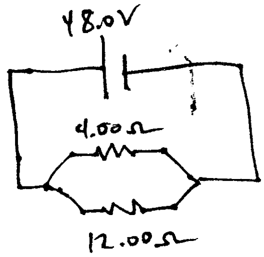
First combine the pairs of series combinations...

1.00Ω 3.00Ω = R_{eff} $\quad R_{eff} = 3.00 + 1.00$

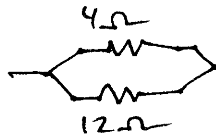
$= \underline{4.00\Omega}$

7Ω 5Ω = $R = 7\Omega + 5\Omega$

$= \underline{12.00\Omega}$

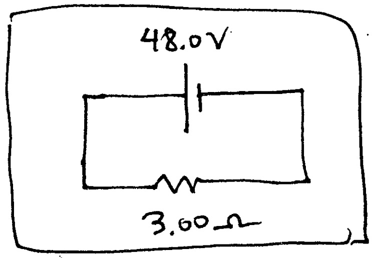


Then combine the parallel combination



$= R = \dfrac{4.00\Omega \cdot 12.00\Omega}{16.00\Omega}$

$= \underline{\underline{3.00\Omega}}$



This is the equivalent resistance network.

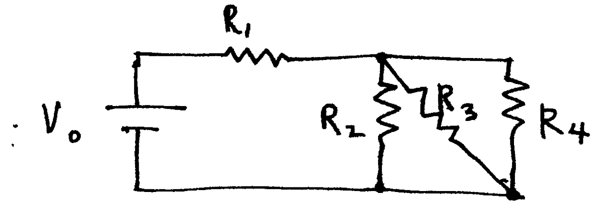
Find the current in each resistor: The total current from the battery is $I_{TOT} = \dfrac{48.0V}{3.00\Omega}$

$I_{TOT} = \boxed{16.0A}$

The current through the series combination of the 1Ω and 3Ω resistors is... $I_1 = \dfrac{48.0V}{4.00\Omega} = \boxed{12.0A}$

And the current through the series combination of the 7Ω and 5Ω resistors is... $I_2 = \dfrac{48.0V}{12.0\Omega} = \boxed{4.0A}$

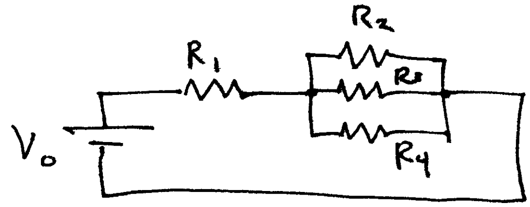
UP 26. $\boxed{10}$ Four resistors and a battery of negligible internal resistance are assembled to make the following circuit.



where $V_0 = 6.00\,V$
$R_1 = 3.50\,\Omega$
$R_2 = 8.20\,\Omega$
$R_3 = 1.50\,\Omega$
$R_4 = 4.50\,\Omega$

(a) Find the equivalent resistance of the network

The above circuit is equivalent to

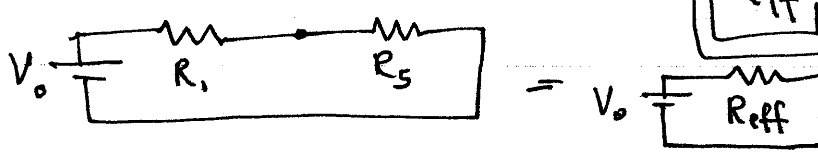


So R_2, R_3 and R_4 are in parallel with effective resistance R_5 such that

$$\frac{1}{R_5} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{8.20\,\Omega} + \frac{1}{1.50\,\Omega} + \frac{1}{4.50\,\Omega}$$

$$= 0.122\,\Omega^{-1} + 0.667\,\Omega^{-1} + 0.222\,\Omega^{-1} = 1.011\,\Omega^{-1}$$

$\boxed{R_5 = 0.99\,\Omega}$ This is in series with R_1 making the equivalent resistance of the whole network... $\boxed{R_{eff} = R_1 + R_5 = 4.49\,\Omega}$



(b) Find the current in each resistor

The total current supplied by the battery is.. $I_{TOT} = \frac{V_0}{R_{eff}} = \frac{6.00\,V}{4.49\,\Omega} = 1.34\,A$

This current must flow through resistor R_1 ... $\boxed{I_1 = 1.34\,A}$ resulting in a voltage drop $V_1 = I_1 R_1 = 1.34\,A \cdot 3.50\,\Omega = 4.68\,V$

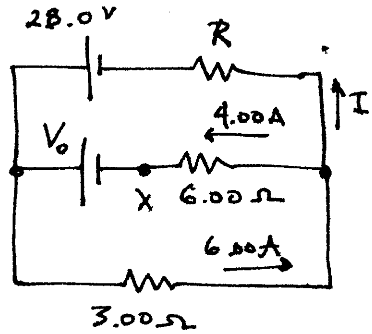
The voltage drop across each of the resistors R_2, R_3 and R_4 is, therefore $V_2 = V_0 - V_1 = 6.00\,V - 4.68\,V = 1.32\,V = V_3 = V_4$

$\boxed{I_2 = \frac{V_2}{R_2}} = \frac{1.32\,V}{8.20\,\Omega} = \boxed{0.16\,A}$ $\boxed{I_3 = \frac{V_3}{R_3}} = \frac{1.32\,V}{1.50\,V} = \boxed{0.88\,A}$

$\boxed{I_4 = \frac{V_4}{R_4}} = \frac{1.32\,V}{4.50\,\Omega} = \boxed{0.29\,A}$ $I_{TOT} = I_2 + I_3 + I_4 = 1.33\,A$ check ✓

UP

26. $\boxed{19}$ In the circuit shown below,



(a) find the current in resistor R.

By Kirchoff's Node (Junction) Rule ...

$$6.00\text{ A} = 4.00\text{ A} + I$$

So $\boxed{I = 2.00\text{ A}}$ in resistor R.

(b) Find the resistance R:

Use Kirchoff's Loop Rule on the outer loop....

$$28.0\text{V} - (6.00\text{ A})(3.00\,\Omega) - (2.00\text{ A})\cdot R = 0$$

$$28\text{V} - 18\text{V} - 2R = 0$$

$$R = \tfrac{1}{2}\cdot 10\text{V} = \boxed{5.00\,\Omega}$$

(c) Find the unknown battery voltage, V_0.

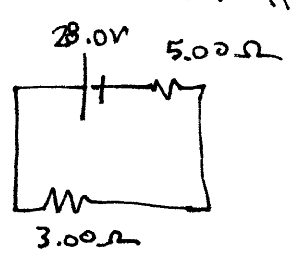
Use Kirchoff's Loop Rule on the lower loop

$$V_0 - (6.00\text{ A})(3.00\,\Omega) - (4.00\text{ A})(6.00\,\Omega) = 0$$

$$V_0 = 18\text{ V} + 24\text{V} = \boxed{42.0\text{ V}}$$

(d) If the circuit is broken at X then what current is in resistor R?

If there is an open pt. in the circuit at X then the middle branch is effectively removed and the effective circuit is..

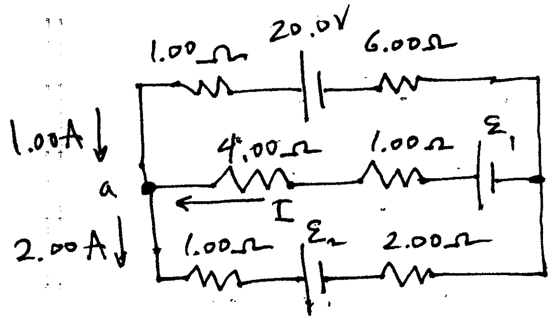


The series combination of 2 resistors has effective resistance $R_{eff} = 3.00\,\Omega + 5.00\,\Omega = 8.00\,\Omega$

And $I = \dfrac{28.0\text{V}}{8.00\,\Omega} = \boxed{3.5\text{ A}}$

UP 26.20

Find the unknown emf's $\mathcal{E}_1$ and $\mathcal{E}_2$ and the potential difference between points a and b ... $V_{ba} = V_b - V_a$.



Using the Kirchoff Junction Rule at point a, allows us to determine the current in the middle branch of the circuit: $\boxed{I = 1.00A}$ so that

the current into a equals the current out of a.

Use the Kirchoff Loop Rule for the top loop to determine $\mathcal{E}_1$:
Going clockwise around starting at point b...

$$\mathcal{E}_1 - 1.00A\cdot1.00\Omega - 1.00A\cdot4.00\Omega + 1.00A\cdot1.00\Omega - 20.V + 1.00A\cdot6.00\Omega = 0$$

going opposite direction to the current

$$\mathcal{E}_1 - 1V - 4V + 1V - 20V + 6V = 0$$

or... $\boxed{\mathcal{E}_1 = 18.0V}$

Use Kirchoff's Loop Rule for the outer loop to determine $\mathcal{E}_2$:
Going clockwise around starting at point b...

$$+2.00A\cdot2.00\Omega + \mathcal{E}_2 + 2.00A\cdot1.00\Omega + 1.00A\cdot1.00\Omega - 20.0V + 1.00A\cdot6.00\Omega = 0$$

$$4V + \mathcal{E}_2 + 2V + 1V - 20V + 6V = 0$$

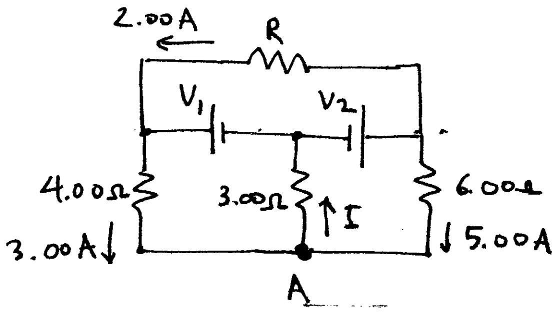
or $\boxed{\mathcal{E}_2 = 7.0V}$

The voltage at b relative to a is found by starting at a and ending at b...

$$V_{ba} = +1.00A\cdot4.00\Omega + 1.00A\cdot1.00\Omega - \mathcal{E}_1$$
$$= 4V + 1V - 18V = \boxed{-13.0V}$$

UP 26.

[21] In the circuit shown below,



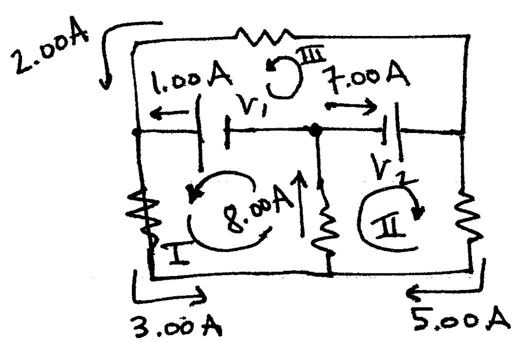
(a) Find the current in the 3Ω resistor

Use Kirchoff's Junction Rule at A

$3.00\,A + 5.00\,A = I = \boxed{8.00\,A}$

(b) Find the unknown battery voltages V_1 and V_2

At this point we know (or can determine) all the branch currents...



Use Kirchoff's Loop Rule on the lower left loop... ... loop I

$$V_1 - (3.00A)(4.00\Omega) - (8.00A)(3.00\Omega) = 0$$

$$V_1 = 12V + 24V = \boxed{36.0V}$$

Now use Kirchoff's Loop Rule on the lower right loop, Loop II

$$V_2 - (5.00A)(6.00\Omega) - (8.00A)(3.00\Omega) = 0$$

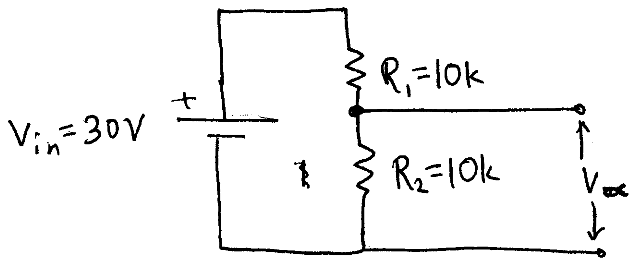
$$V_2 = 30V + 24V = \boxed{54.0V}$$

(c) Find the resistance, R: Use Kirchoff's Loop Rule on the top loop - (loop III).

$$V_2 - (2.00A)R - V_1 = 0$$

$$R = \frac{V_2 - V_1}{2.00A} = \frac{54V - 36V}{2.00A} = \frac{18V}{2A} = \boxed{9.00\Omega}$$

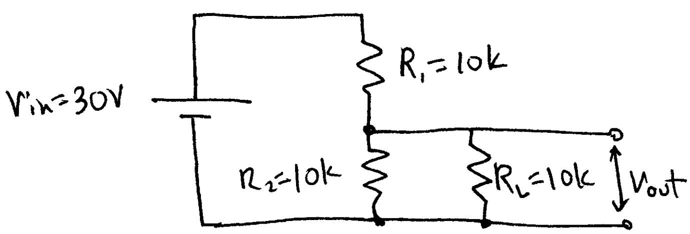
A+H $\boxed{1.9}$ a) Open circuit voltage (no load on voltage divider circuit)



Using the voltage divider relation (p.8)

$$V_{oc} = \frac{R_2}{R_1 + R_2} V_{in} = \frac{10k}{20k} 30V = \boxed{15V}$$

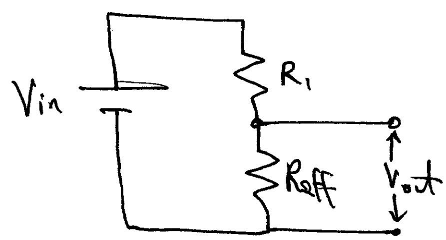
b) The output voltage with a 10k load.



First replace R_2 and R_L with parallel combination

$$R_{eff} = \frac{R_2 R_L}{R_2 + R_L} = \frac{10k \cdot 10k}{20k} = 5k$$

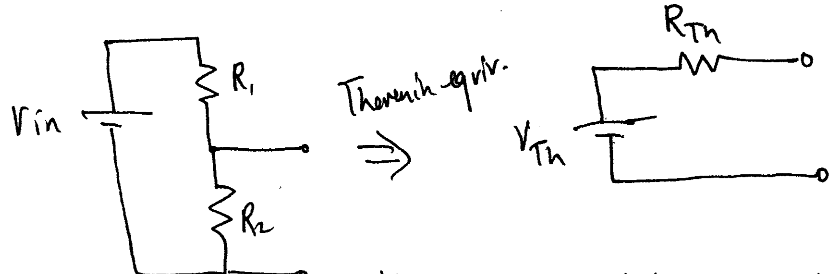
Now the circuit looks like a voltage divider



$$V_{out} = \frac{R_{eff}}{R_1 + R_{eff}} V_{in}$$

$$= \frac{5k}{10k + 5k} \cdot 30V = \boxed{10V}$$

c) The Thevenin equivalent voltage for the voltage divider is the open circuit voltage from part a)



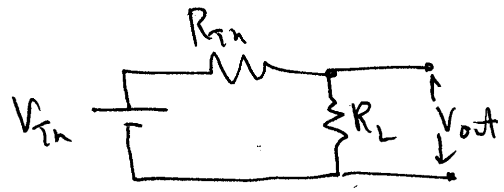
$$\boxed{V_{Th} = 15V}$$

If the output is shorted the current is $I_{sc} = \frac{V_{in}}{R_1} = \frac{30V}{10k}$

$= \underline{3mA}$

and $R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{15V}{3 \times 10^{-3} A} = \boxed{5k\Omega}$

d) If a 10k load is attached then the Thevenin circuit with load is a voltage divider..

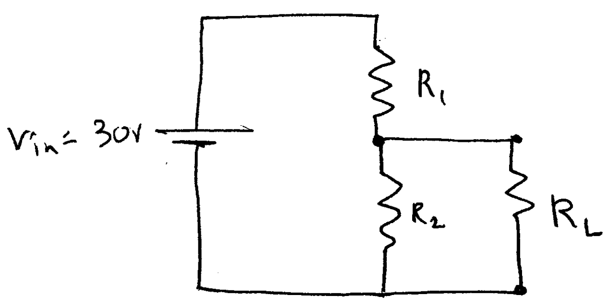


$$V_{out} = \frac{R_L}{R_L + R_{Th}} V_{Th}$$

$$= \frac{10k}{10k + 5k} 15V = \boxed{10V}$$

Same as part (b).

e) Power dissipated in each resistor



There is 10V across the load so...

$$P_{Load} = \frac{V_{out}^2}{R_{Load}} = \frac{(10V)^2}{10k} = \boxed{0.01\,W}$$

There is also 10V across R_2, since it is in parallel with R_L. So, since $R_2 = 10k$

$\boxed{P_2 = 0.01W}$ also

There is 20V across R_1 (since 30V battery − 10V across bottom resistor, leaves 20V).

So... $P_1 = \frac{(20\,V)^2}{10k} = \boxed{0.04\,W}$