4-15 Calculate the three longest wavelengths in the Lyman series ($n_f = 1$) in nm. Indicate the series limit. Are any of these lines in the visible spectrum?

Hydrogen spectral lines are well-described by $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ where $R = 1.097\times10^7\ m^{-1}$

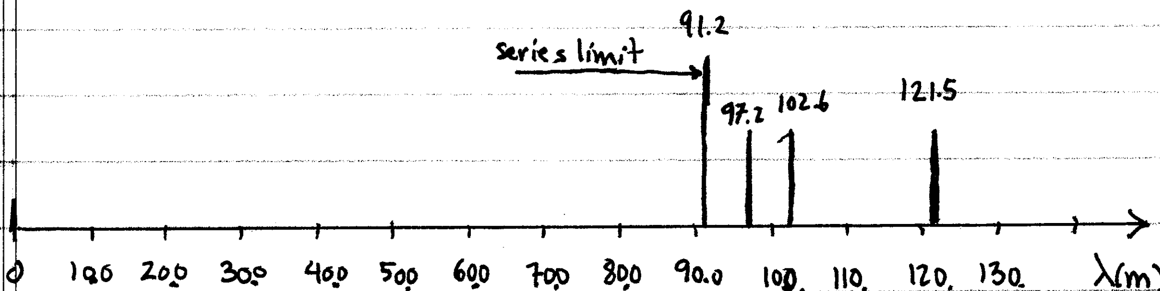
The three longest wavelengths correspond to the three smallest energy transitions...

(1) $n_i = 2 \rightarrow n_f = 1$ $\quad \frac{1}{\lambda(m)} = 1.097\times10^7\ m^{-1}\left[\frac{1}{1} - \frac{1}{4}\right] \rightarrow \lambda(nm) = \boxed{121.5\ nm}$

(2) $n_i = 3 \rightarrow n_f = 1$ $\quad$ " $\left[\frac{1}{1} - \frac{1}{9}\right] \rightarrow \boxed{102.6\ nm}$

(3) $n_i = 4 \rightarrow n_f = 1$ $\quad$ " $\left[1 - \frac{1}{16}\right] \rightarrow \boxed{97.2\ nm}$

The series limit is $n_i = \infty \rightarrow n_f = 1$ $\quad$ " $\left[1 - \frac{1}{\infty}\right] \rightarrow \boxed{91.2\ nm}$



None of these lines is in the visible spectrum which is ~400 nm → ~700 nm

4-19 Muonic hydrogen atom. Mass of muon 105.7 MeV/c^2 compared to the mass of the electron, 0.511 MeV/c^2.

(a) Calculate the radius of the first Bohr orbit of a muonic atom.

→ The derivation of the Bohr model is identical except m_e is replaced by m_μ.

So the Bohr radius is...

(Eq. 4-19) $$a_0 = \frac{\hbar^2}{m_e k e^2} \Rightarrow a_\mu = \frac{\hbar^2}{m_\mu k e^2}$$

Or...

$$a_\mu = \frac{\hbar^2}{m_e k e^2} \cdot \frac{m_e}{m_\mu} = \left(\frac{m_e}{m_\mu}\right) a_0$$

$$a_\mu = \left(\frac{0.511}{105.7}\right) \cancel{0.529 Å}\, a_0 = \frac{a_0}{206.8} = \frac{0.0529\,\text{nm}}{206.8}$$

$$a_\mu = 2.56\times10^{-4}\,\text{nm} = \boxed{2.56\times10^{-13}\,\text{m}}$$ ~~= 25.6 fm~~

$$= 256\,\text{fm}$$

(b) Calculate the magnitude of the lowest energy.

From Eq. 4-20 $$E_0 = \frac{m_e k^2 e^4}{2\hbar^2} \Rightarrow E_{\mu_0} = \frac{m_\mu k^2 e^4}{2\hbar^2}$$

Or.. $$E_{\mu,0} = \frac{m_e k^2 e^4}{2\hbar^2} \cdot \left(\frac{m_\mu}{m_e}\right) = 206.8 \cdot E_0$$

$E_0 = 13.6\,\text{eV}$

$$\boxed{E_{\mu,0} = 2812\,\text{eV} = 2.812\,\text{keV}}$$

(c) What is the shortest wavelength in the Lyman series for this atom?

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$ the Rydberg constant is $R = \frac{E_0}{hc}$

so... $$R_\mu = \left(\frac{m_\mu}{m_e}\right) R = 206.8 \cdot 1.097\times10^7\,\text{m}^{-1}$$

$$= 2.269\times10^9\,\text{m}^{-1}$$

So.. $n_i = 2 \rightarrow n_f = 1$, the longest Lyman wavelength is

$$\frac{1}{\lambda} = 2.269\times10^9\,\text{m}^{-1}\left[1 - \frac{1}{4}\right] \rightarrow \boxed{\lambda = 5.88\times10^{-10}\,\text{m} = 0.588\,\text{nm} = 5.88\,Å}$$

4-23 What is the radius of the $n=1$ orbit in C^{5+}? What is the energy of the electron in that orbit? What is the wavelength of the radiation emitted by C^{5+} in the Lyman α transition?

The Bohr model derivation is the same for C^{5+} except $Z=6$ .. the charge on the nucleus of this one electron atom.

So... using eq. 4-18 $\quad r_n = \frac{n^2 a_0}{Z}$ $\quad$ for $n=1$, $Z=6$...

$$r_1 = \frac{a_0}{6} = \frac{0.0529\,\text{nm}}{6} = \boxed{0.00882\ \text{nm}}$$

The energy is given ~~by~~ eq. 4-20

$$E_n = -\frac{Z^2 E_0}{n^2} \qquad \text{so, for } n=1,\ Z=6..$$

$$E_1 = -\frac{6^2 E_0}{1} = -36 \cdot 13.6\,\text{eV} = \boxed{490\ \text{eV}}$$

$$= \boxed{7.83 \times 10^{-17}\ \text{J}}$$

The Lyman α transition is $n_i = 2$ to $n_f = 1$... so..

eq. 4-22 $\quad \frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

$$\frac{1}{\lambda} = 36 \cdot 1.097\times10^{7}\,\text{m}^{-1}\left(1 - \frac{1}{4}\right) \rightarrow \lambda = 3.38\times10^{-9}\,\text{m}$$

$$\boxed{\lambda = 3.38\ \text{nm}}$$

This is in the extreme ultraviolet or soft X-ray region of the electromagnetic spectrum.

$\boxed{4\text{-}25}$ Rydberg hydrogen atom with $n \approx 600$

(a) What would be the diameter of a hydrogen atom with $n \approx 600$?

eq. 4-18 $\quad r_n = a_0 n^2$

$$= a_0 (600)^2$$

$$= 19{,}000 \text{ nm}$$

$$\boxed{r_n = 19.0\,\mu\text{m} = 1.90\times10^{-5}\text{ m}}$$

(b) What would be the speed of the electron in that orbit ?

From angular momentum conservation...

$$mvr_n = n\hbar$$

or $v = \dfrac{n\hbar}{mr_n} = \dfrac{n\hbar}{ma_0n^2} = \dfrac{\hbar}{ma_0 n}$

$$= \frac{1.05\times10^{-34}\text{ Js}}{(9.11\times10^{-31}\text{ kg})(0.0529\times10^{-9}\text{ m})(600)}$$

$$v_{100} = \frac{2.18\times10^{6}\text{ m/s}}{600} = \boxed{3.63\times10^{3}\text{ m/s} = 3.63\text{ km/s}}$$

(c) The speed for the $n=1$ orbit is 600 times higher..

$$\underline{\underline{v_1 = 2.18\times10^{6}\text{ m/s}}}$$

5-2 Find the deBroglie wavelength of a relativistic electron ... $E = 100$ MeV.

Relativistically $E = \sqrt{p^2c^2 + m^2c^4} \approx pc$ when $E \gg mc^2$

So... $p = \frac{E}{c}$ or... $\frac{h}{\lambda} = \frac{E}{c}$

DeBroglie relation for p..

Solve for the wavelength $\lambda = \frac{hc}{E}$

and, using $hc = 1240$ eV·nm ... $\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{100\times10^6 \text{ eV}}$

$= 1.240\times10^3 \cdot 10^{-8}$ nm

$= 1.24\times10^{-5}$ nm

$$\boxed{\begin{aligned}\lambda &= 1.24\times10^{-14} \text{ m} \\ &= 12.4 \text{ fm}\end{aligned}}$$

$1 \text{ fm} = 10^{-15}$ m

5-3

Electrons in an electron microscope are accelerated through a potential difference of V_o so that their wavelength is 0.04nm. What is V_o?

Use the deBroglie relation to find the momentum.. $p = \frac{h}{\lambda}$

Then ... assuming non-relativistic electrons, the kinetic energy is...

$$E_k = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

This will be equal to eV_o ... so determine E_k, in electron-Volts

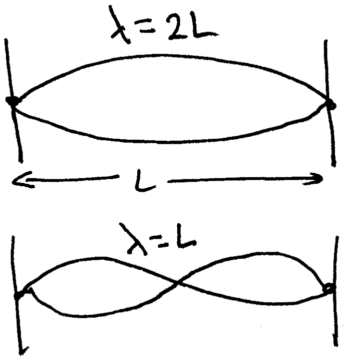
$$E_k = \frac{(hc)^2}{2(mc^2)\lambda^2} = \frac{(1240\ eV\cdot nm)^2}{2(511{,}000\ eV)(0.04\ nm)^2} = 940\ eV$$

So... $\boxed{V_o = 940\ \text{Volts}}$

since this is much less than the rest mass energy of the electron, we are justified in using non-relativistic equations.

5-7 A free proton moves back and forth between rigid walls separated by a distance $L = 0.01\ nm$.

(a) If the proton is represented by a one-dimensional standing deBroglie wave with a node at each wall, show that the allowed values of the deBroglie wavelength are given by $\lambda = \frac{2L}{n}$.



This is just the same as the standing waves on a string tied down at both ends..

There must be an integral number of half wavelengths in L ...

$$L = n\left(\frac{\lambda}{2}\right) \quad \text{or...} \quad \boxed{\lambda = \frac{2L}{n}}$$

(b) Derive a general expression for the allowed kinetic energy...

Since $p = \frac{h}{\lambda}$ and $E_k = \frac{p^2}{2m}$ (non-relativistic)

$$E_k = \frac{h^2}{2m\lambda^2} \quad \text{substitute in } \lambda = \frac{2L}{n}..$$

$$\boxed{E_k = \frac{h^2}{2m} \cdot \frac{n^2}{4L^2}} = \boxed{\frac{h^2}{8mL^2} \cdot n^2}$$

For $n=1$ $\quad E_k = \frac{h^2}{8mL^2} = \frac{(6.626\times10^{-34} Js)^2}{8(1.67\times10^{-27} kg)(0.01\times10^{-9} m)^2}$

$$= \boxed{3.29\times10^{-19}\ J = 2.05\ eV}$$

For $n=2$, the result is 2^2 or 4 times larger

$$\boxed{1.31\times10^{-18} J = 8.18\ eV}$$

$\boxed{5\text{-}24}$

Wave function for a particle in a 1D box of length L: $\psi = A\sin\left(\frac{\pi x}{L}\right)$

Since the particle must be somewhere in the box, the total (or integrated) probability must be 1.

$$\int_0^L \underbrace{|\psi|^2}_{\text{probability density}} dx = 1$$

So...
$$\int_0^L A^2 \sin^2\left(\frac{\pi x}{L}\right) dx = 1$$

use trig identity $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

$$A^2 \int_0^L \frac{1}{2}\left(1-\cos\left(\frac{2\pi x}{L}\right)\right) dx = 1$$

$$A^2\left\{\frac{1}{2}\int_0^L dx - \frac{1}{2}\int_0^L \cos\left(\frac{2\pi x}{L}\right) dx\right\} = 1$$

$$A^2\left[\frac{L}{2} - \frac{1}{2}\underbrace{\left[\frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^L}_{\text{zero at both limits}}\right] = 1$$

$$A^2\frac{L}{2} = 1$$

or... $\boxed{A = \sqrt{\frac{2}{L}}}$

5-27

If an excited state of an atom is known to have a lifetime of 10^{-7} s, what is the uncertainty in the energy of photons emitted by such atoms in spontaneous decay to the ground state?

Use the energy-time uncertainty principle $\Delta E \Delta t \gtrsim \hbar/2$

where $\Delta t = \tau$ the lifetime of the state

So... $\Delta E \approx \frac{\hbar}{2\tau} = \frac{6.582\times10^{-16}\,eV\cdot s}{2\cdot10^{-7}s} = \underline{\underline{3.3\times10^{-9}\,eV}}$

or $= \underline{\underline{5.27\times10^{-28}\,J}}$

5-30

If the uncertainty in the position of a wave packet representing the state of a quantum system particle is equal to its deBroglie wavelength, how does the uncertainty in momentum compare with the value of the momentum?

So, let $\Delta x = \underbrace{\frac{h}{p}}_{\text{deBroglie wavelength}}$

Then, from the uncertainty principle... $\Delta p \gtrsim \frac{\hbar}{2\Delta x} \approx \frac{\hbar p}{2h}$

or $\boxed{\Delta p \gtrsim \frac{p}{4\pi}}$ The uncertainty in the momentum is at least 8% $\left(\frac{1}{4\pi}\right)$ of the value of its momentum.

$\boxed{5\text{-}32}$ Locate an electron to within $\Delta x = 5\times10^{-12}$ m using light with this wavelength.

The momentum of a photon with $\lambda = 5\times10^{-12}$ m is...

$$p_\gamma = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\,\text{J s}}{5\times10^{-12}\,\text{m}} = 1.33\times10^{-22}\ \text{kg}\frac{\text{m}}{\text{s}}$$

The uncertainty in the momentum of the electron, from the uncertainty principle, is...

$$\Delta p\,\Delta x \geq \frac{\hbar}{2} \quad \text{so...}$$

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{1.055\times10^{-34}\,\text{J s}}{2\cdot 5\times10^{-12}\,\text{m}} = 1.06\times10^{-23}\ \text{kg}\frac{\text{m}}{\text{s}}$$

$$\frac{\hbar}{2\Delta x} \downarrow \frac{\lambda}{4\pi}$$

$$= \underline{8\% \text{ of } p_\gamma}$$

$\boxed{5\text{-}39}$ The lifetime of the Δ resonance particle computed from the energy width 250 MeV.

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

$\Delta t = \tau$ lifetime of the particle ...

$$\tau \geq \frac{\hbar}{2\,\Delta E} = \frac{\hbar c}{2c\,\Delta E} = \frac{197\ \text{eV}\cdot\text{nm} \times 10^{-9}\ \text{m/nm}}{2\cdot(3\times10^{8}\ \frac{\text{m}}{\text{s}})(250\times10^{6}\ \text{eV})}$$

$$\tau \gtrsim \boxed{1.3\times10^{-24}\ \text{s}}$$

extremely short time!

$\boxed{5\text{-}40}$ Neutrons and protons in the nucleus are confined to a region of size $\Delta x \simeq 10^{-15}$ m

(a) At any given instant, how fast might an individual proton or neutron be moving?

Use the uncertainty principle ... $\Delta p \Delta x \geq \hbar/2$

where $\Delta p = m \Delta v$ ... so

$$\Delta v \geq \frac{\hbar}{2 m \Delta x} = \frac{1.05\times10^{-34}\,\text{J}\cdot\text{s}}{2\cdot 1.67\times10^{-27}\,\text{kg}\cdot 10^{-15}\,\text{m}}$$

$$= 3.1\times10^{7}\,\text{m/s}$$ or ~ 10% of the speed of light.

(b) What is the kinetic energy of a neutron in the nucleus?

From eq. 5-28

$$E \geq \frac{\hbar^2}{2m_n L^2} = \frac{(\hbar c)^2}{2(m_n c^2)\cdot L^2} = \frac{(197\,\text{eV}\cdot\text{nm})^2}{2(\cancel{938}\times10^{6}\,\text{eV})(10^{-6}\,\text{nm})^2}$$

940

$\uparrow 10^{-15}\,\text{m} = 10^{-6}\,\text{nm}$

So... $E \gtrsim 2.1\times10^{7}\,\text{eV} = \boxed{21\,\text{MeV}}$

↑ ~~still~~ much less than rest mass energy, justifying use of non-relativistic equations

(c) What would be the corresponding energy of an electron localized to within such a region?

The electron's kinetic energy would be larger than for the neutron (or proton) by a factor of...

$$\frac{m_n}{m_e} = \cancel{\frac{938}{0.511}} = \frac{940}{0.511} = 1840$$

So $E_{electron} \gtrsim \underline{3.9\times10^{10}\,\text{eV}}$ ← very relativistic ... so equations we used are not really appropriate.