13 Coordinates of event in S: $x = 75\,m$, $y = 18\,m$, $z = 4.0\,m$, $t = 2.0\times10^{-5}\,s$

Frame S′ moves in +x direction at $v = 0.85c$ and coincides with S at $t = t' = 0$

(a) What are coordinates of event in S′? Use Lorentz txn equations:

$$x' = \gamma(x - vt), \quad y' = y, \quad z' = z, \quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = 1.89832$ .. keep lots of digits (6) for now, round off later.

$$x' = 1.89832\,(75 - 0.85\cdot 2.9979\times10^{8}\,m/s \cdot 2.0\times10^{-5}\,s) = -9532.3\,m$$

$$\boxed{x' = -9500\,m}$$

$$\boxed{y' = 18\,m}$$

$$\boxed{z' = 4.0\,m}$$

$$t' = 1.89832\left(2.0\times10^{-5}\,s - 0.85\cdot\frac{75\,m}{2.9979\times10^{8}\,m/s}\right) = 3.75627\times10^{-5}\,s$$

$$\boxed{t' = 3.8\times10^{-5}\,s}$$

(b) Use inverse transformations to get original coordinates.... need to use more precise (unrounded) numbers.

$$x = \gamma(x' + vt'), \quad y = y', \quad z = z', \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$\gamma = 1.89832$ still

$$x = 1.89832\left(-9532.3 + 0.85\cdot 2.9979\times10^{8}\,m/s \cdot 3.7563\times10^{-5}\,s\right) = \underline{75\,m}\ ✓$$

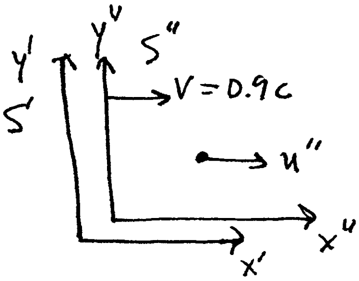
using numbers rounded to 2 sig. figs. $x = 1.9\,(-9500 + 0.85\cdot 3\times10^{8}\,m/s\cdot 3.8\times10^{-5}\,s) = \underline{360\,m}$ way off!

$y = y' = \underline{18\,m}$ $\qquad$ $z = z' = 4.0\,m$

$$t = 1.89832\left(3.7563\times10^{-5}\,s + 0.85\cdot\frac{-9532.3\,m}{2.9979\times10^{8}\,m/s}\right) = 2.0\times10^{-5}\,s\ ✓$$

19 A particle moves with speed 0.9c along x'' axis in S'' which moves at 0.9c in $+x'$ direction relative to S'. S' moves at 0.9c in $+x$-direction relative to S.

(a) Find the speed of the particle relative to S'

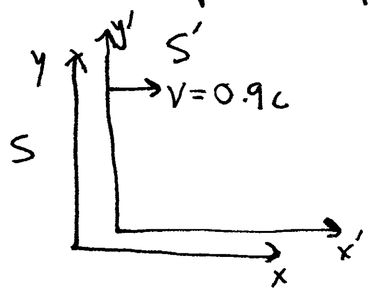


Need to use the inverse velocity transformation equations (Eqs 1-23 on p. 22); with primes → double primes
unprimed → primed coords.

$$u_x' = \frac{u_x'' + V}{\left(1 + \frac{V u_x''}{c^2}\right)} = \frac{0.9c + 0.9c}{1 + \frac{0.9c \cdot 0.9c}{c^2}} = \frac{1.8c}{1 + 0.81}$$

$$\underline{\underline{u_x' = 0.9945c}}$$

(b) Find the speed of the particle relative to S.



Same process..

$$u_x = \frac{u_x' + V}{\left(1 + \frac{V u_x'}{c^2}\right)} = \frac{0.9945c + 0.9c}{1 + \frac{0.9c \cdot 0.9945c}{c^2}}$$

$$= \frac{1.8945c}{1.8950} = \boxed{0.9997c}$$

$\boxed{25}$ Measured length of spacecraft $L = 85\,m$. Proper length is $100\,m$.
What is speed?

Use length contraction formula $L = L_p\sqrt{1 - v^2/c^2}$

Solve for speed... $\left(\frac{L}{L_p}\right)^2 = 1 - v^2/c^2$

$$\frac{v^2}{c^2} = 1 - \left(\frac{L}{L_p}\right)^2 \quad \text{or} \quad v/c = \sqrt{1 - \left(\frac{L}{L_p}\right)^2}$$

$$\frac{v}{c} = \sqrt{1 - (0.85)^2} = \boxed{0.53}$$

33 The red light emitted by hydrogen 656.3 nm.

If the source is receding from earth with speeds... (a) $v = 10^{-3}c$, (b) $10^{-2}c$, (c) $10^{-1}c$

Doppler effect for source receding: $f = \sqrt{\dfrac{1-\beta}{1+\beta}}\, f_0$

where $\beta = \dfrac{v}{c}$ $\quad f_0 = \dfrac{c}{\lambda_0}$ $\quad$ and $f = \dfrac{c}{\lambda}$

So...
$$\frac{c}{\lambda} = \sqrt{\frac{1-\beta}{1+\beta}} \cdot \frac{c}{\lambda_0}$$

or..
$$\lambda = \sqrt{\frac{1+\beta}{1-\beta}}\, \lambda_0$$

(a)
$$\lambda = \sqrt{\frac{1+10^{-3}}{1-10^{-3}}} \cdot 656.3\,\text{nm} = 1.001 \cdot 656.3\,\text{nm} = \underline{\underline{657.0\,\text{nm}}}$$

(b)
$$\lambda = \sqrt{\frac{1+10^{-2}}{1-10^{-2}}} \cdot 656.3\,\text{nm} = 1.01 \cdot 656.3\,\text{nm} = \underline{\underline{662.9\,\text{nm}}}$$

(c)
$$\lambda = \sqrt{\frac{1+10^{-1}}{1-10^{-1}}} \cdot 656.3\,\text{nm} = 1.106 \cdot 656.3\,\text{nm} = \underline{\underline{725.6\,\text{nm}}}$$

$\boxed{34}$ A version of the twin paradox:

- Heide leaves earth and travels a constant velocity $0.45c$ toward Betelgeuse.
- One year later, twin brother, Hans leaves and travels at $0.95c$

(a) When Hans catches Heide, what will be the difference in their ages?

First find how much time elapses on earthbound clocks by the time Hans catches up with Heide.

Heide's position $\quad x_1 = v_1 t \quad$ where $v_1 = 0.45c$

Hans's position $\quad x_2 = \begin{cases} 0 & t < 1\,yr = t_0 \\ v_2(t - t_0) \end{cases}$

Find time when $x_1 = x_2$

$$v_1 t = v_2(t - t_0)$$

$$t(v_2 - v_1) = v_2 t_0$$

$$t = \frac{v_2 t_0}{(v_2 - v_1)} = \frac{0.95c \cdot 1yr}{(0.95c - 0.45c)} = \frac{0.95\,l.y.}{0.5c} = \underline{1.9\,yrs}$$

- The time elapsed on Heide's clock is the proper time interval... Invert the time dilation formula. $\quad \tau_1 = \Delta t_1 \sqrt{1 - v_1^2/c^2}$

$$= 1.9yrs\sqrt{1 - (0.45)^2} = \underline{1.7\,yrs}$$

Hans ages one year on earth before departing. Another 0.9 yrs elapse on earthbound clocks but Hans experiences proper time interval...

$$\tau_2 = \Delta t_2 \sqrt{1 - v_2^2/c^2} = 0.9yrs\sqrt{1 - (0.95)^2} = 0.28\,yrs..$$

So Hans ages $\underline{1.28yrs}$ while Heide ages $\underline{1.7yrs}$. Difference is $\underline{0.42}$ yrs.

(b) $\underline{\text{Hans}}$ is $\underline{\text{younger}}$. $\underline{\text{Heide}}$ is $\underline{\text{older}}$.

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In frame S, event B occurs $2\mu s$ after event A and at $\Delta X = 1.5$ km from event A.

(a) How fast must an observer be moving along $+X$ axis so that events A and B occur simultaneously?

In frame S' $\Delta t' = 0$ for events A and B. Use Lorentz transformation equations to find velocity of S' relative to S.

$$\Delta t' = \gamma\left(\Delta t - \frac{V\Delta X}{c^2}\right) = 0$$

So... $\Delta t - \frac{V\Delta X}{c^2} = 0$ ~~s~~solve for V..

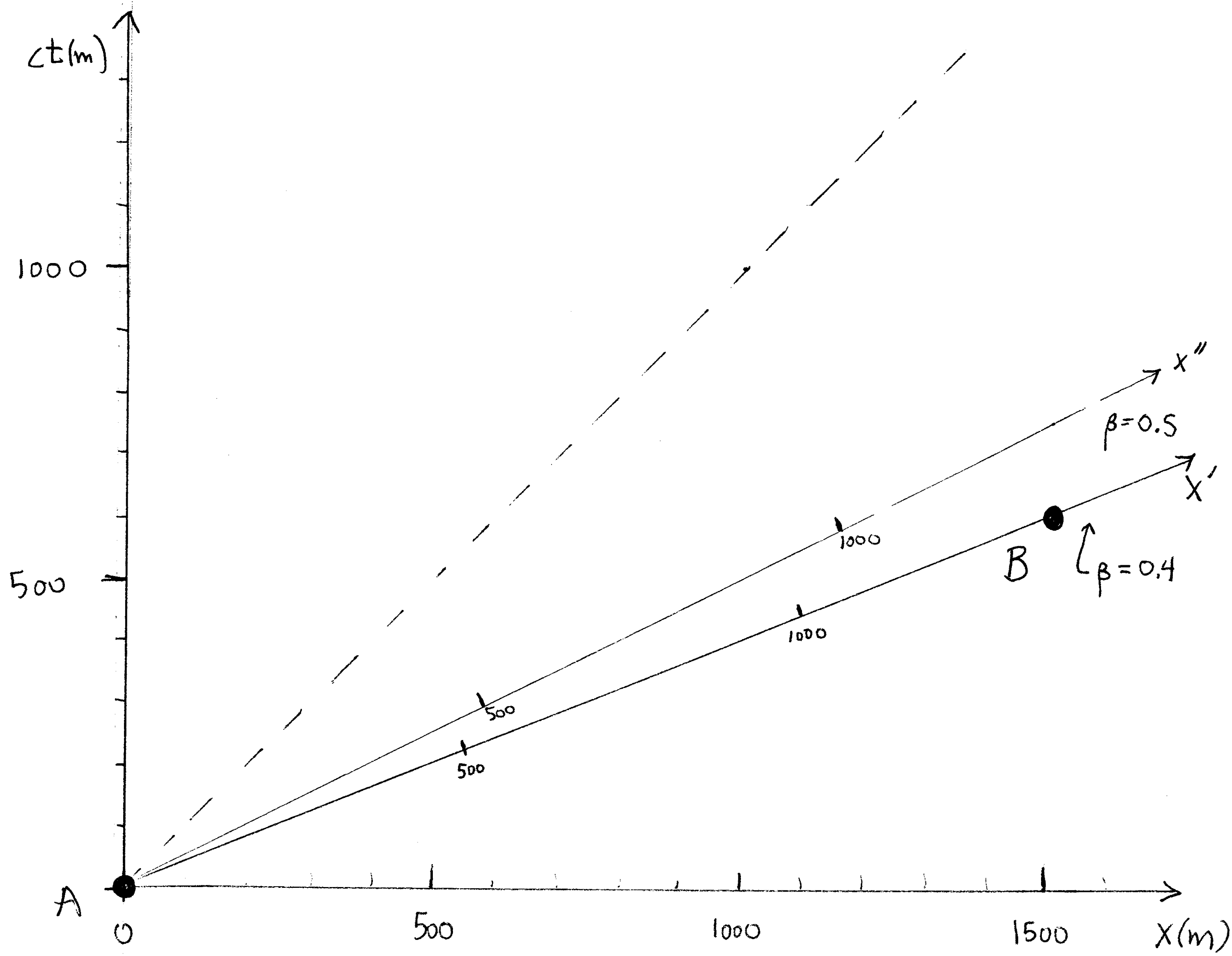
$$V = \frac{\Delta t\, c^2}{\Delta X} = \frac{2\times10^{-6}\,s\,(3\times10^{8}\,m/s)^2}{(1.5\times10^{3}\,m)} = 12\times10^{7}\ m/s = \underline{1.2\times10^{8}\,m/s}$$

or.. $\boxed{V = 0.4c}$

(b) Is it possible for event B to precede event A for some observer?

YES. A and B are SPACELIKE separated events. In frame S, A occurs before B. If we go to a frame moving in $+X$ direction relative to S at speed $V = 0.4c$, then A and B are simultaneous. If $V > 0.4c$, then $\Delta t' < 0$ and B precedes A in that frame.

(c) See next page. $c\Delta t = 3\times10^{8}\,m/s \cdot 2\times10^{-6}\,s = 600\,m$



Slope of x' axis projected onto spacetime diagram for S is β.
In frame S', A and B lie on x' axis (at $\Delta t' = 0$). In frame S'' ($\beta = 0.5$)
B lies <u>below</u> the x'' axis, indicating that B occurs <u>before</u> A in that frame

(d) Compute spacetime interval : $(\Delta S)^2 = (c\Delta t)^2 - \Delta x^2$

$$= (600\,\text{m})^2 - (1500\,\text{m})^2 = -1.89 \times 10^6\,\text{m}^2$$

The same in any reference frame. So the <u>minimum distance</u> between events A and
↳ "proper distance"
is the distance measured in a frame where A and B are simultaneous ($\Delta t' = 0$)...

$$\Delta X_{min} = \sqrt{-(\Delta S)^2} = \sqrt{1.89 \times 10^6\,\text{m}} = \underline{\underline{1375\,\text{m}}}$$

45 Observers in S see an explosion at $x_1 = 480\,m$. A second explosion occurs $5\mu s$ later at $x_2 = 1200\,m$. In S' the two explosions occur at the same point in space.

(a) Spacetime diagram is on next page. In S, the separation of the two events along the ct axis is...

$$c\Delta t = 3\times10^8\,m/s \cdot 5\times10^{-6}\,s = 1500\,m$$

(b) Since the two events occur at the same location in S', one of the clocks in S' is present for both events and a line connecting the two events has a slope equal to c/v.

$$\text{slope} = \frac{c\Delta t}{\Delta x} = \frac{1500\,m}{(1200\,m - 480\,m)} = 2.08$$

$$\text{so} \quad \frac{v}{c} = \frac{1}{2.08} = 0.48 \qquad \boxed{v = 0.48\,c}$$

(c) Calibrate the ct' axis and determine the separation in time of the two events in S'.

Use the inverse Lorentz transformation equation

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$\text{or} \quad ct = \gamma(ct' + \beta x')$$

We want the ct values for various ct' values at $x' = 0$ along the ct' axis... so...

$$ct = \gamma(ct')$$

$$\text{for } ct' = 1000\,m \qquad ct = \frac{1000\,m}{\sqrt{1-(0.48)^2}} = \underline{\underline{1140\,m}}$$

From the spacetime diagram the separation in time of the two events in S' is

$$c\Delta t' = 1050\,m - (-260\,m) = 1310\,m$$

$$\Delta t' = \frac{1310\,m}{3\times10^8\,m/s} = \boxed{4.4\,\mu s}$$

(d) Use the Lorentz transformation equations to verify the result in part (c).

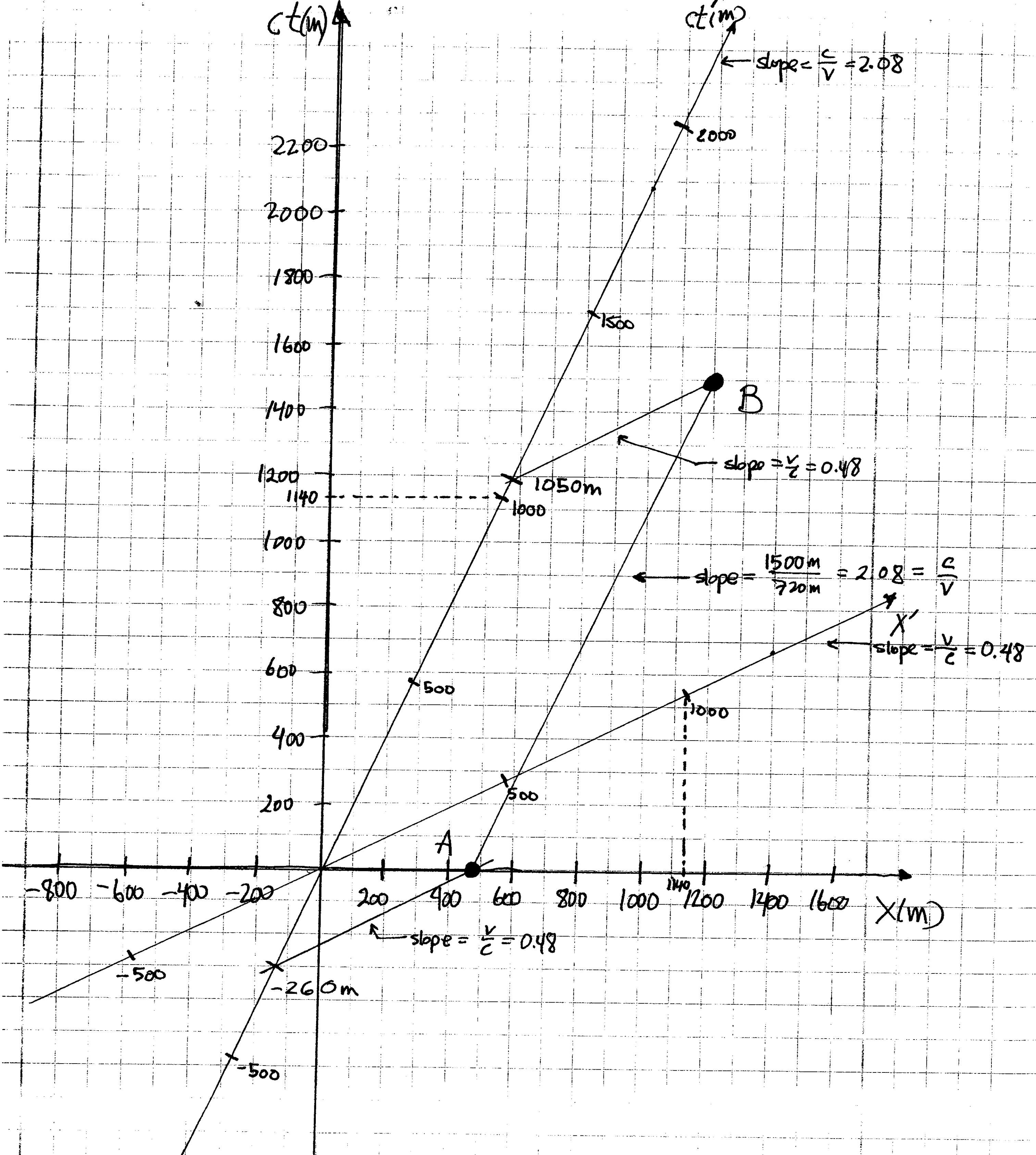
$$c\Delta t' = \gamma\,(c\Delta t - \beta\Delta x)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.14 \qquad \beta = 0.48$$

$c\Delta t = 1500\ \text{m}$ and $\Delta x = 720\ \text{m}$

$$c\Delta t' = 1.14\,(1500\ \text{m} - 0.48 \cdot 720\ \text{m}) = \underline{1316\ \text{m}}$$

$$\underline{\underline{\Delta t' = 4.4\ \mu s}}$$



ct(m)
ct'(m)
slope = c/v = 2.08
2000
2200
2000
1800
1500
1600
B
1400
slope = v/c = 0.48
1200
1050m
1140
1000
1000
slope = 1500m/720m = 2.08 = c/v
800
X'
slope = v/c = 0.48
600
500
1000
400
500
200
A
-800
-600
-400
-200
200
400
600
800
1000
1140
1200
1400
1600
X(m)
slope = v/c = 0.48
-500
-260m
-500

$\boxed{54}$ Equation for a spherical wavefront of light that begins at the origin at $t=0$.

$$x^2+y^2+z^2-(ct)^2=0 \qquad \text{in } S$$

Use the Lorentz transformation equations to show that

$$\underline{x'^2+y'^2+z'^2-(ct')^2=0} \qquad \text{in } S' \quad \text{also....}$$

Given: $\quad x^2+y^2+z^2-(ct)^2=0$

and inverse Lorentz txn eqs. $\quad x=\gamma(x'+vt')\ ,\ y=y',\ z=z',\ t=\gamma\left(t'+\frac{vx'}{c^2}\right)$

substitute into equation for spherical wavefront...

$$[\gamma(x'+vt')]^2+y'^2+z'^2-c^2\left[\gamma\left(t'+\frac{vx'}{c^2}\right)\right]^2=0$$

multiply out...

$$\gamma^2(x'^2+2vx't'+v^2t'^2)+y'^2+z'^2-c^2\gamma^2\left(t'^2+\frac{2v}{c^2}x't'+\frac{v^2x'^2}{c^4}\right)=0$$

collect terms...

$$\underbrace{x'^2\left[\gamma^2-\frac{c^2\gamma^2v^2}{c^4}\right]}_{①}+\overbrace{x't'\left[\gamma^2 2v-\frac{2c^2\gamma^2v}{c^2}\right]}^{②}+y'^2+z'^2$$

$$+\underbrace{t'^2\left[\gamma^2v^2-c^2\gamma^2\right]}_{③}=0$$

① $\gamma^2-\frac{v^2}{c^2}\gamma^2=\gamma^2\left(1-\frac{v^2}{c^2}\right)=\frac{(1-v^2/c^2)}{(1-v^2/c^2)}=1$

② $\gamma^2 2v-\gamma^2 2v\frac{c^2}{c^2}=0$

③ $\gamma^2v^2-\gamma^2c^2=\gamma^2c^2\left(\frac{v^2}{c^2}-1\right)=-c^2\frac{(1-v^2/c^2)}{(1-v^2/c^2)}=-c^2$

So...

$$x'^2\cdot 1+x't'\cdot 0+y'^2+z'^2+t'^2(-c^2)=0$$

or $\quad x'^2+y'^2+z'^2-c^2t'^2=0 \quad$ <u>Same as in S</u> !