DE HW #5 selected solutions

4.2:18) <u>claim</u> For a vector space V and a scalar k, $k \cdot \emptyset = \emptyset$

<u>pf</u>

By definition of zero vector,

$$\emptyset + \emptyset = \emptyset$$

multiply by k:

$$k(\emptyset + \emptyset) = k\emptyset$$

distribute:

$$k\emptyset + k\emptyset = k\emptyset$$

add $-k\emptyset$:

$$(k\emptyset + k\emptyset) + -k\emptyset = k\emptyset + -k\emptyset$$

reassociate

$$k\emptyset + (k\emptyset + -k\emptyset) = k\emptyset + -k\emptyset$$

by definition of $-k\emptyset$:

$$k\emptyset + \emptyset = \emptyset$$

by definition of $\emptyset$:

$$k\emptyset = \emptyset \quad \checkmark$$

4.3: (8) (a) $\begin{bmatrix} 6 & 3 \\ 0 & 8 \end{bmatrix} = k_1 \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + k_2 \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix} + k_3 \begin{bmatrix} 4 & -2 \\ 0 & -2 \end{bmatrix}$

So we have
$$6 = k_1 + 4k_3$$
$$3 = 2k_1 + k_2 - 2k_3$$
$$0 = -k_1 + 2k_2$$
$$8 = 3k_1 + 4k_2 - 2k_3$$

Thus, $k_1 = 2$, $k_2 = 1$, and $k_3 = 1$, so the matrix $\begin{bmatrix} 6 & 3 \\ 0 & 8 \end{bmatrix}$ is a linear combination of A, B, and C.

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = k_1 \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + k_2 \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix} + k_3 \begin{bmatrix} 4 & -2 \\ 0 & -2 \end{bmatrix}$

So $k_1 = k_2 = k_3 = 0$ satisfies the equation. Thus the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a linear combination of A, B, and C.

4.3: (10) (a) $\cos 2x$ lies in the space spanned by $\sin^2 x$ and $\cos^2 x$ if we can find constants k_1 and k_2 that satisfy the equation $k_1 \cos^2 x + k_2 \sin^2 x = \cos 2x$.

If we use the trig identity $\cos 2x = \cos^2 x - \sin^2 x$, we have $k_1 \cos^2 x + k_2 \sin^2 x = \cos^2 x - \sin^2 x$.

So $k_1 = 1$ and $k_2 = -1$. Thus, $\cos 2x$ lies in the space spanned by $\sin^2 x$ and $\cos^2 x$.

(c)

We need to find constants k_1 and k_2 that satisfy the equation $k_1 \sin^2 x + k_2 \cos^2 x = 1$.

We know that $\sin^2 x + \cos^2 x = 1$, thus $k_1 = k_2 = 1$ satisfies the equation, so 1 lies in the space spanned by $\sin^2 x$ and $\cos^2 x$.

25. Since y_1 and y_2 are solutions, they are differentiable. The hypothesis can thus be restated as $y_1'(t_0) = y_2'(t_0) = 0$ at some point t_0 in the interval of definition. This implies that $W(y_1, y_2)(t_0) = 0$. But $W(y_1, y_2)(t_0) = c\,exp(-\int p(t)dt)$, which *cannot* be equal to zero, unless $c = 0$. Hence $W(y_1, y_2) \equiv 0$, which is ruled out for a fundamental set of solutions.