

**Midterm Exam**  
**Differential Equations**  
**Fall 2005**  
**Hunsicker**

Notes: Make sure you explain your work clearly and completely. I can't give partial credit if I don't understand what you are doing. Try to annotate your calculations as much as possible. There are 150 points possible on this exam. You will be graded out of 100 points. You may attempt up to 110 points. **Choose which 110 points of problems you wish to be graded on and indicate them clearly!** You will have 2 hours to complete it. Allot your time accordingly. You may use a calculator on this exam, but if you do, indicate how you have used it (except for basic calculations). No other aids are permitted, such as books, notes or classmates.

Theory

1. (5 points) Define exact o.d.e.
2. (10 points) Define bifurcation point.
3. (5+5 points) Induction
  - a. State the Principle of Induction
  - b. Use induction to show that the  $n^{\text{th}}$  derivative of the function  $f(x) = x^n$  is equal to the constant function  $f^{(n)}(x) = n!$ .
4. (15 points) Explain what Euler's Method is used to do, and use a picture to explain how it works.
5. (10 points) State the Existence and Uniqueness Theorem for First Order ODE's.
6. (20 points) State and prove the General Solution Theorem (Theorems 3.2.3 and 3.2.4).

Word Problem (If you are totally stuck, I will give you the answers to a and b at the cost of 5 points so you can do parts c and d).

7. (20 points) A certain piece of dubious information about phenylethylamine in the drinking water began to spread one day in a city with a population of 100,000. Within a week, 10,000 people had heard this rumor.
  - a. Let  $y(t)$  = the number of people who have heard the rumor by day  $t$ . Use this to write an expression for the number of people in the town who have NOT heard the rumor by day  $t$ .
  - b. Assume the rate at which new people hear the rumor is proportional to the product of the number of people who have not yet heard the rumor times the number of people who have heard the rumor (this is an *interaction term*, since each new person to hear it hears it from some person who heard it previously). Write a differential equation for  $y$ .
  - c. Assume that the rumor started with one person at time  $t=0$ . Solve the IVP and find the constant of proportionality in the equation in part b. (Hint: You know the value of  $y(t)$  at a time other than  $t=0$  as well).
  - d. How long will it be before half the city has heard the rumor?

Calculations

8. (10 points) Show that the function  $y=(1 + x^2)^{-1}$  is a solution to the equation

$$y' + 2xy^2 = 0.$$

9. (15 points) Draw a slope field diagram and several possible solution curves for the equation  $y' = x^2 + y^2 - 4$ . Make your diagram LARGE so I can read it.

10. (10+5+5 points) Consider the equation  $y'' - y' - 6y = 0$ .

a. Find a fundamental set of solutions to this problem and write down the general solution.

b. Find the solution to the initial value problem that has initial conditions  $y(0) = 2, y'(0) = -1$ .

c. Show that if  $y_1(t) = e^{\pi t}$  and  $y_2(t) = e^{st}$  are two solutions to a constant coefficient linear homogeneous o.d.e and  $s \neq \pi$ , then they always form a fundamental set of solutions.

11. (15 points) Solve the equation  $y' - y = (11/8) e^{-x/3}$ .