Euler's Method calculations using the TI 89

Ex 2 Using the TI 89,

Use Euler's Method to solve

$$y' = \frac{y^2 + 2ty}{3 + t^2} \qquad y(0) = \frac{1}{2}$$

at $t = 1$

a) using steps of size .1 (ie, 10 of them)

b) using steps of size .01 (ie, 100 of them)

c) If we want accuracy to .005, what step size do we seem to need?

On your TI 89

1) Press MODE and set Graph = DIFF EQUATIONS
2) PRESS ◆ F1 (F6 Y=). Enter y1' = (y1^2 + 2·t·y1)/(3+t^2)
   yi1 = $\frac{1}{2}$
3) PRESS ◆ I (F). Set Solution Method = Euler
   Fields = FLDOFF
4) PRESS ◆ F2 (F7 WINDOW). Set t0 = 0
   tmax = 1
   tstep = .1
   tplot = 0
   xmin = -1   ncurves = 0
   xmax = 100   Estep = 1
   xscl = 1
   ymin = -10
   ymax = 10
   yscl = 1

NOTES 1) $t0 = t_0$
$tmax = t_1$
$\frac{tmax - t0}{tstep} \times Estep = n$ = total # steps

2) You can change $\begin{cases} t0, \\ y1' = \\ yi1 = \end{cases}$ to get a different IVP
then change $tplot = t_0$, too.

IF the equation degenerates, you may have to restrict xmin, xmax, ymin, ymax.

5) Press $\boxed{\text{HOME}}$. PRESS $\boxed{\text{CATALOG}}$. PRESS $\boxed{\text{alpha}}$ $\overset{\{\ \ B}{\boxed{(}}$.
Scroll to BldData. Press $\boxed{\text{ENTER}}$.

6) Press $\boxed{\text{2nd}}$ $\boxed{\text{alpha}}$. Type euler. Press $\boxed{\text{alpha}}$.
Press $\boxed{\text{enter}}$

7) Press $\boxed{\text{APPS}}$ $\boxed{6}$ $\boxed{3}$. Type "errorlog" in the variable place and enter it twice

8) Above c1 type "time"
above c2 type "Euler"

9) enter c1 = euler[1]
c2 = euler[2]

You should get a table that looks like this:

Data	t c1	y1 c2	
1	0.	.5	$= y_0$
2	.1	.50833	$= y_1$
⋮	.2	.5203	$= y_2$
⋮	⋮	⋮	⋮
11	1.	.76799	$= y_{10}$, the approximation given by a step size of h=. Record this value.

10) Press $\boxed{\text{HOME}}$, then $\boxed{\blacklozenge}$ $\overset{\text{F7 WINDOW}}{\boxed{\text{F2}}}$. Change Estep to 10.
This gives:
$$\text{Estep} \times \frac{1}{\text{Estep}} \times (t_1 - t_0) = 10 \times 10 \times 1 = 100 \text{ steps}$$
or $h = .01$

11) Press $\boxed{\text{HOME}}$, then re-enter BldData euler.

$$h = \frac{\text{tstep}}{\text{Estep}}$$

12) Press [apps] [6] [1]

13) Now the table has been rebuilt for h=.01 steps. It should look like:

Data	t c1	y1 c2
1	0	.5
2	.1	.50998
⋮	⋮	⋮
11	1.	.7966

(note once you have built this table once, you don't have to do it again -- it will readjust if you change step size ~~[illegible]~~)

14) Repeat steps 10)-13), but in step 10), set Estep = 100. This is h=.001 in Euler's method. Your table should now look like:

Data	t c1	y1 c2
1	0	.5
2	.1	
⋮		
11	1.	.79966

15) Since the first two digits have stabilized, we probably now have an answer accurate to .005.

Solving the equation exactly (when possible)

16) Now press $\boxed{\text{HOME}}$, then $\boxed{\text{F3}}$.
Scroll down to DEsolve and press $\boxed{\text{enter}}$.

17) DeSolve (y' = (y^2 + 2*t*y)/(3+t^2), t, y)

the solution is

$$y = \frac{-(t^2+3)}{t - @1}$$

NOTE
@1 is a variable, like our C.

18) Press $\boxed{\text{F2}}$, and enter "solve"
19) Press $\boxed{\blacktriangle}$, $\boxed{\text{ENTER}}$ } to bring solution down. Replace y with .5, @1 with x, t with 0,
to get "solve (.5= -(0^2 +3) / (0-x), x)"
add this

and enter this.
You should get x=6.

20) Scroll up again to $y = \frac{-(t^2+3)}{t - @1}$ and press $\boxed{\text{enter}}$

Replace t with 1, @1 with 6 and erase "y=".

Press $\boxed{\text{enter}}$.

You should get $\frac{4}{5}$, which is .8, as our exact answer. So our approximation, .79966, was, in fact, good to within .005.

Generally, of course, you can't do 17-20, and just have to guess based on the approximations.