

Differential Equations with Linear Algebra
Midterm Exam
Fall 2003

You will have two hours to complete this exam. Point totals are given by each problem, so allot your time accordingly. You may use a calculator, but show all work. No notes or books are permitted. There are 110 points possible. The exam will be graded out of 90.

- 1) (10 points) State the existence and uniqueness theorem for first order ode's.
- 2) Exact Equations
 - a) (5 points) State the exact equations theorem.
 - b) (10 points) Prove this theorem.
- 3) A simplified model for the progress of AIDS in an individual is given by the equation $V' = \lambda V (I - \alpha V)^2$, where $V(t)$ = amount of virus present in the body at time t , and λ , I , and α are positive constants.
 - a) (6 points) Find the steady state solutions, draw the slope field diagram, and describe the stability of the steady state solutions. You need not solve the equation generally.
 - b) (5 points) One of the goals of this model was to explain the long latency period observed in HIV infected individuals, that is, the long period during which an individual may be infected without exhibiting symptoms of AIDS. How could this model account for a latency period?
 - c) (4 points) The variable I denotes the immune response of the individual, and is fairly constant when the individual is basically healthy. What happens to the steady states and slope field diagram if I changes? How could this model explain how full blown AIDS may develop in a previously asymptomatic individual?
- 4) A 600-gallon tank is filled with 300 gallons of pure water. A spigot is opened above the tank and a salt solution containing 1.5 lb of salt per gallon of solution begins flowing into the tank at a rate of 3 gallons/minute. Simultaneously, a drain is opened at the bottom of the tank allowing the solution to leave the tank at a rate of 1 gallon/minute.
 - a) (3 points) Draw a diagram of the situation, labeling all relevant quantities.
 - b) (5 points) Set up a differential equation for $Q(t)$ = pounds of salt in the tank at time t . Notice that the volume of the tank is not constant.
 - c) (7 points) How many pounds of salt are in the tank when it is exactly at capacity?
- 5) (10 points) Write a short essay explaining Euler's method of approximation.
- 6) (5 points) Define basis and dimension of a vector space.
- 7) (5 points) State the existence and uniqueness theorem for linear second order ode's.

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8) Homogeneous linear equations:

- a) (5 points) Give the book's definition of homogeneous linear second order ode
- b) (5 points) Give the in class definitions of homogeneous linear differential equation.
- c) (5 points) Show that any equation satisfying the book's definition also satisfies the class definition.

9) a) (5 points) Find a pair of solutions to $y'' + 8y' - 9y = 0$.

b) (5 points) Find the solution to the IVP:

$$y'' + 8y' - 9y = 0$$

$$y(0) = 12$$

$$y'(0) = -8$$

10) a) (6 points) Show that $y_1 = \cos(\ln t)$ and $y_2 = \sin(\ln t)$ are solutions to the equation $t^2 y'' + t y' + y = 0$.

b) (4 points) Show they form a basis for the solution space for this equation.