Quiz #4

Calculus 160

Spring 2006, Hunsicker

Name KEY IHRTLUHC

1) Prove that if r (t) parametrizes a path on a sphere centered around the origin, then its tangent vector is perpendicular to its position vector for all t.

If $\vec{r}(t)$ is on a sphere around the origin then $\vec{r}(t)$ = position and $\vec{r}\,'(t)$ = tangent. If radius of sphere = k then $|\vec{r}(t)| = K$. Thus $\vec{r}(t)\cdot\vec{r}(t) = |\vec{r}(t)|^2 = K^2$

so $\frac{d}{dt}[\vec{r}(t)\cdot\vec{r}(t)] = \frac{d}{dt}[k^2] = 0$. But by dot. prod rule,

$\vec{r}\,'(t)\cdot\vec{r}(t) + \vec{r}(t)\cdot\vec{r}\,'(t) = 0$ and by commut. of $\bullet$,

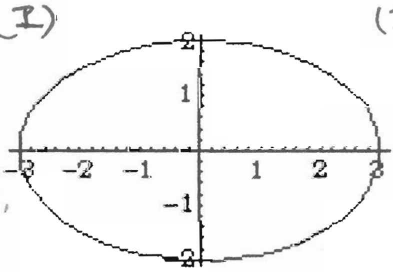
$2\vec{r}\,'(t)\cdot\vec{r}(t) = 0$ so dividing by 2,

$\vec{r}\,'(t)\cdot\vec{r}(t) = 0$, ie $\vec{r}\,'(t) \perp \vec{r}(t)$.

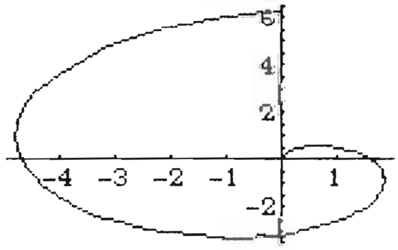
2) Match the parametric equations to the curves. Explain your choices:

a) < 3 sin(t), 2 cos(t) > (I) b) <t sin(t), t cos(t)> (II) c) < sin(t), cos (t^2) > (III) d) <cos(t), 1+sin(t)> (IV)

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ is the equation of this ellipse, (a) satisfies it

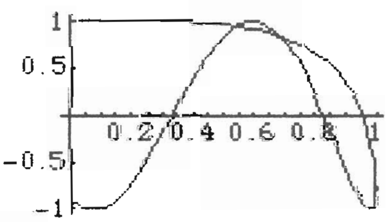


I)

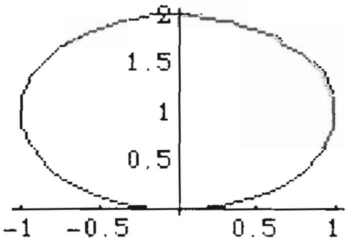


II)

This is a spiral, ie, radius of "circle" increases with t - this is <t sint, t cost



III)



IV)

This is the only graph to pass through (0,1), as does <sin(t), cos(t^2)> at t = 0.

<cost, 1+sint> is the unit circle moved up by 1 on the y-axis