# PRACTICE MIDTERM #2

I. a) State the heat equation.

   b) __Essay__ Explain why this equation is the right one to describe heat flow along a rod.

II. a) Define normal vector to a surface

   b) Prove that $\nabla f$ is a normal vector to the level surfaces of $f(x,y,z)$.

III. Estimate the % error made in calculating the kinetic energy, $K(m,v) = \frac{1}{2}mv^2$, of a car travelling along a highway if your measurement of mass is accurate to within 5% and your measurement of velocity is accurate to within 2%.

IV. a) Draw the graph of the function $f(x,y) = -\ln(x^2+y^2)$

   b) Calculate the directional derivative of $f$ in the direction $\vec{w} = (1,2)$ at the point $(0,-1)$. Explain the meaning of this number using your drawing from a).

V. Suppose a ball of radius 1 has temperature given by the function $T(x,y,z) = xyz$. Use the method of Lagrange multipliers to find the hottest and coldest spots on the surface of the ball and the temperatures at those spots.

VI. a) Define the limit at $(a, b)$ of a function $f(x, y)$.

b) Use the definition ($\varepsilon - \delta$ proof) to show that $\lim\limits_{(x, y) \to (4, 3)} x + y + \sqrt{x^2 + y^2} = 12$.