PRACTICE MIDTERM #1

I. a) Define cross product of two vectors in $\mathbb{R}^3$.
b) Prove $\vec{v} \times \vec{w}$ is perpendicular to both $\vec{v}$ and $\vec{w}$
c) Describe $\vec{v} \times \vec{w}$ geometrically
d) Find $\vec{v} \times \vec{w}$ for $\vec{v} = (1,2,3)$, $\vec{w} = (2,0,1)$

II. a) Draw $\vec{v} = (1,3)$ and $\vec{w} = (4,2)$
b) Find $\vec{v} + \vec{w}$. Draw it. Find $\vec{v} - \vec{w}$. Draw that.
c) Draw $\text{proj}_{\vec{w}} \vec{v}$. Calculate $\text{proj}_{\vec{w}} \vec{v}$.

III. a) Find the volume of the parallelepiped given by the vectors $\vec{u} = (1,1,-2)$, $\vec{v} = (0,3,1)$, $\vec{w} = (-4,2,1)$
b) Find the distance of $P = (2,1)$ from the line l given by $6x - 8y + 3 = 0$

IV. a) Define parametrization of a curve
b) sketch the curve associated to $F(t) = (t\cos t, t\sin t)$
c) Find the arclength of this curve between $t = 0$ and $t = 2\pi$
d) Find the velocity of a particle whose position is given by $F(t)$. Sketch the vector at $t = \pi/2$.
e) what happens to the speed of the particle as $t \longrightarrow \infty$?
f) Find the acceleration of the particle at $t = \pi/2$. Draw it.
g) Find the parametric equation of the tangent line to this curve at $t = \pi/4$. Draw it.

IV. a) Define curvature of a curve
b) Draw a picture to explain this definition
c) Use a formula to find the curveture for the curve given by $F(t) = (t, t^2)$
d) what does this say about the curve?