

Calculus 160
Spring 2004
Prof. Hunsicker

MIDTERM EXAM

This is a two hour exam. No notes or books or neighbors are permitted as help. You may use a calculator. Please write legibly. The more clearly you document your thought process on a problem, the more credit you are likely to receive. If you have any questions, come ask me. There are 100 points possible, as indicated for each problem. Allot your time accordingly.

Definition and Theorem Statements: (5 points each)

- 1) Define the vector space \mathbb{R}^n .
- 2) Define smooth vector valued function and smooth curve.
- 3) State the theorem on tangential and normal components of acceleration.

Theorem Proofs

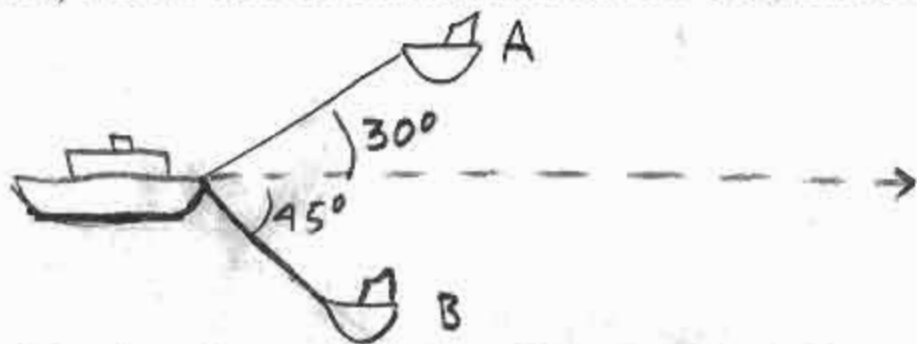
- 4) (10 points) State and prove the derivative formula for dot products of vector valued functions.
- 5) (15 points) Prove that a 3 x 3 determinant can be interpreted as a volume.

Essay

- 6) (12 points) Explain how matrix equations generalize scalar equations. Make clear how the various properties of matrices come in at each step.

Word Problems (10 points each)

- 7) Two tugboats pull an ocean freighter as shown in the diagram below. If tugboat A exerts a force of 1000 pounds, with what force must tugboat B pull to ensure the freighter moves along the indicated dotted line? In this case, what is the resultant force on the freighter in that direction?



- 8) Projectile motion on Mars: On Mars, which has virtually no atmosphere, and where wind resistance is therefore negligible, the acceleration due to gravity is about 12 feet/sec². If a ball is thrown from a height of 6 feet at a 45 degree angle with an initial speed of 50 feet/second, find the function describing its trajectory. How far will it go?

Calculations

9) (6 points each) Consider the vector valued function

$$f(t) = \langle \cos(t), \sin(t) + 1, 3t \rangle.$$

- a) Find the parametric equation for the tangent line to the curve of $f(t)$ at the point $(\sqrt{3}/2, 3/2, \pi/2)$.
- b) Calculate the curvature of $f(t)$ at this point.
- c) Calculate the length of the curve between the points $(1, 1, 0)$ and $(\sqrt{3}/2, 3/2, \pi/2)$.

10) (10 points) Find the point of intersection of the planes defined by the equations

$$x + 2y + 3z = 2$$

$$2x + 5y + 3z = -1$$

$$x + 8z = 0$$