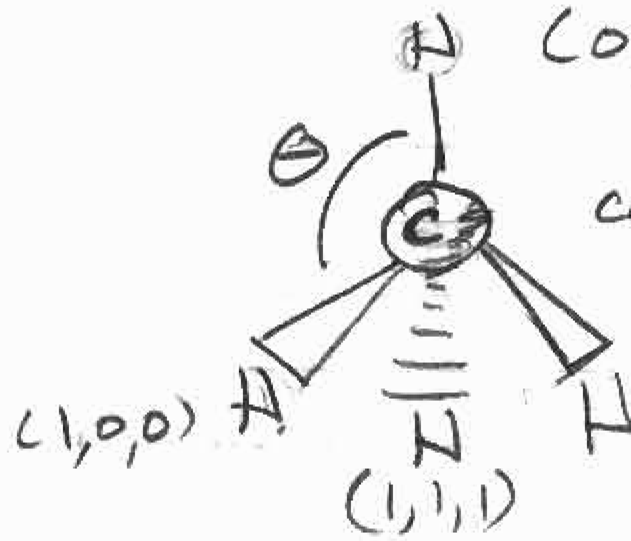
Selected solutions
Homework # 2

12.3 53)



Hydrogens are at vertices

carbon is at centroid $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$

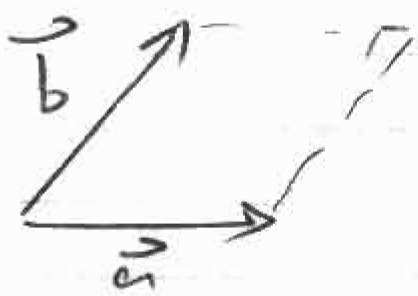
The bond angle, Θ is the angle formed by two hydrogens bonded to the carbon.
So it is the angle between the vectors $\langle \frac{1}{2}-0, \frac{1}{2}-0, \frac{1}{2}-1 \rangle$ and $\langle \frac{1}{2}-1, \frac{1}{2}, \frac{1}{2} \rangle$, eg

$$\cos\Theta = \frac{\langle \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \rangle \cdot \langle -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle}{\|\langle \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \rangle\| \|\langle -\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \rangle\|}$$

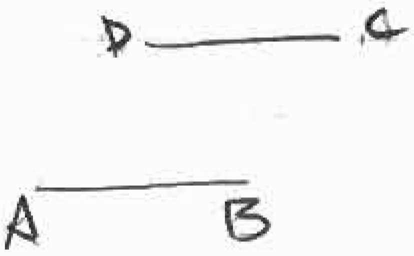
$$= \frac{-1/4}{3/4} = -\frac{1}{3}$$

So $\Theta = \arccos(-\frac{1}{3}) \approx 109.5^\circ$

12.4 23) The area of the parallelogram is $\left\| \begin{vmatrix} c_1 & c_2 \\ c_3 & c_4 \end{vmatrix} \right\|$ where $\langle c_1, c_2 \rangle$ and $\langle c_3 c_4 \rangle$ span it.
So we need to know which points are adjacent to a given vertex, eg, to the first one $(-2,1)$



$\overrightarrow{AB} = \langle 2,3 \rangle$
$\overrightarrow{DC} = \langle 2,3 \rangle$ } these are equal, so they must be opposite sides.



So $\langle c_1, c_2 \rangle = \overrightarrow{AB} = \langle 2,3 \rangle$
$\langle c_3, c_4 \rangle = \overrightarrow{AD} = \langle 4,-2 \rangle$

$$\left\| \begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} \right\| = |2\cdot -2 - 4\cdot 3| = |-4 - 12|$$
$$= 16$$

12.4 45) suppose $a \neq 0$.

a) If $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ does it follow $\vec{b} = \vec{c}$?

No - it just means $\vec{b}, \vec{c}$ are both $\perp \vec{a}$ but, eg, we could have

$$\vec{a} = \vec{i}, \quad \vec{b} = \vec{j}, \quad \vec{c} = \vec{k}.$$

Then this is true but $\vec{b} \neq \vec{c}$.

b) If $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$ does $\vec{b} = \vec{c}$?

No, eg, they could both be parallel to $\vec{a}$. Then both sides $= \vec{0}$, but, eg we could have $\vec{b} = 2\vec{a}$
$\vec{c} = -3\vec{a}$.

c) If $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b} = \vec{a}\times\vec{c}$ does $\vec{b} = \vec{c}$?

Yes. Then subtracting, we get

$$\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c} = \vec{a}\cdot(\vec{b}-\vec{c}) = 0, \text{ so } \vec{b}-\vec{c} \perp \vec{a}$$

and $\vec{a}\times\vec{b} - \vec{a}\times\vec{c} = \vec{a}\times(\vec{b}-\vec{c}) = 0$, so $\vec{b}-\vec{c} \parallel \vec{a}$.

But the only way a single vector can be both $\perp \vec{a}$ and $\parallel \vec{a}$ is if it $= \vec{0}$, so $\vec{b}-\vec{c} = \vec{0}$, ie $\vec{b} = \vec{c}$.

<u>12.5</u> 12) Find the line of intersection of the planes $x+y+z=1$ and $x+z=0$.

$x+y+z=1$ is $\perp$ to $\vec{n}_1 = \langle 1,1,1 \rangle$
$x+z=0$ is $\perp$ to $\vec{n}_2 = \langle 1,0,1 \rangle$

so ℓ is $\perp$ to both, ie $\ell \parallel \langle 1,1,1\rangle \times \langle 1,0,1 \rangle$

Also, solving simultaneously:

$$\left.\begin{array}{r} x+y+z=1 \\ -\quad x+0+z=0 \\ \hline y=1. \end{array}\right\} \Rightarrow \begin{array}{l} x+1+z=1 \\ x+0+z=0 \end{array} \Rightarrow x+z=0$$

Let $x=0$. Then $z=0$ and $y=1$ so $(0,1,0) \in \ell$.

So ℓ passes through $(0,1,0)$ and is parallel to $\langle 1,1,1\rangle \times \langle 1,0,1\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = i\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= i - k = \langle 1, 0, -1 \rangle$$

so ℓ is given by $\vec{r} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.