

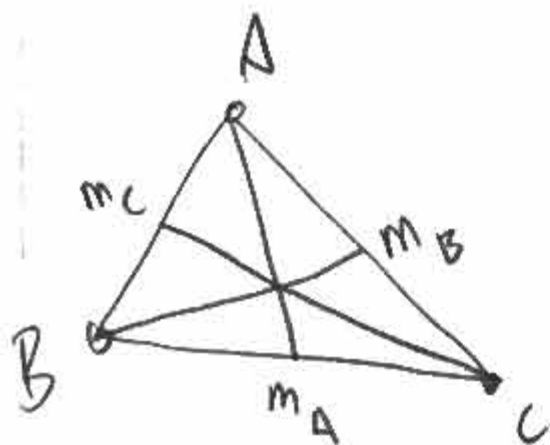
Calc 160 Spring 2005  
HW #1 selected solutions

12.1 19) a) Prove that the midpoint of the line segment from  $P_1(x_1, y_1, z_1)$  to  $P_2(x_2, y_2, z_2)$  is  $m = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2} \right)$ .

There are several ways to do this problem. We will do it by showing that  $\overrightarrow{P_1 m} = \frac{1}{2} \overrightarrow{P_1 P_2}$ . This implies the vectors have the same direction, i.e., the points  $P_1, m, P_2$  are collinear, and that the distance from  $P_1$  to  $m$  is  $\frac{1}{2}$  the distance from  $P_1$  to  $P_2$ .

$$\begin{aligned} \overrightarrow{P_1 m} &= \left\langle \left( \frac{x_1+x_2}{2} \right) - x_1, \left( \frac{y_1+y_2}{2} \right) - y_1, \left( \frac{z_1+z_2}{2} \right) - z_1 \right\rangle \\ &= \left\langle \frac{x_2-x_1}{2}, \frac{y_2-y_1}{2}, \frac{z_2-z_1}{2} \right\rangle = \frac{1}{2} \langle x_2-x_1, y_2-y_1, z_2-z_1 \rangle \\ &= \frac{1}{2} \overrightarrow{P_1 P_2}. \end{aligned}$$

b) Find the lengths of the medians of the triangle with vertices  $A(1, 2, 3)$ ,  $B(-2, 0, 5)$  and  $C(4, 1, 5)$



medians are  $\overline{m_C C}$ ,  $\overline{m_B B}$  and  $\overline{m_A A}$ .

$$\begin{aligned} m_A &= \text{midpoint of } BC = \left( \frac{-2+4}{2}, \frac{0+1}{2}, \frac{5+5}{2} \right) \\ &= \left( 1, \frac{1}{2}, 5 \right) \end{aligned}$$

$$m_B = \text{midpt of } AC = \left( \frac{1+4}{2}, \frac{2+1}{2}, \frac{3+5}{2} \right) = \left( \frac{5}{2}, \frac{3}{2}, 4 \right)$$

$$m_C = \text{midpt of } AB = \left( \frac{1+(-2)}{2}, \frac{2+0}{2}, \frac{3+5}{2} \right) = \left( -\frac{1}{2}, 1, 4 \right)$$

lengths of medians are:

$$|m_A A| = \text{dist}((1, \frac{1}{2}, 5), (1, 2, 3))$$

$$= \sqrt{(1-1)^2 + (\frac{1}{2}-2)^2 + (5-3)^2} = \sqrt{0 + \frac{9}{4} + 4}$$
$$= \sqrt{\frac{25}{4}} = \frac{5}{2}$$

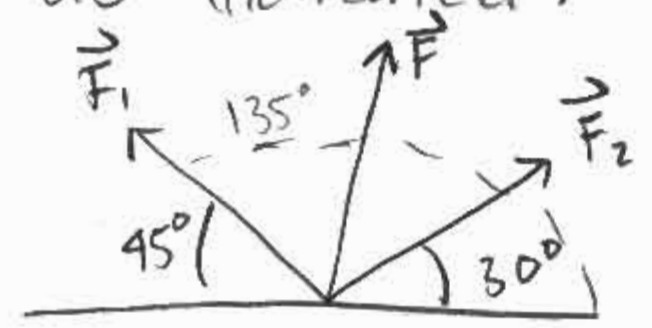
$$|m_B B| = \text{dist}((\frac{5}{2}, \frac{3}{2}, 4), (-2, 0, 5))$$

$$= \sqrt{(\frac{5}{2}-(-2))^2 + (\frac{3}{2}-0)^2 + (4-5)^2}$$
$$= \sqrt{\frac{81}{4} + \frac{9}{4} + 1} = \sqrt{\frac{94}{4}} = \frac{\sqrt{94}}{2}$$

$$|m_C C| = \text{dist}((-\frac{1}{2}, 1, 4), (4, 1, 5))$$

$$= \sqrt{(-\frac{1}{2}-4)^2 + (1-1)^2 + (4-5)^2}$$
$$= \sqrt{\frac{81}{4} + 0 + 1} = \sqrt{\frac{85}{4}} = \frac{\sqrt{85}}{2}$$

29)  $F_1$  has magnitude 10 lb,  $F_2$  has magnitude 12 lb as indicated:



Find magnitude & direction of resultant force,  $\vec{F}$ .

$$\vec{F}_2 = \langle 12 \cos 30^\circ, 12 \sin 30^\circ \rangle = \langle 6\sqrt{3}, 6 \rangle$$
$$+ \vec{F}_1 = \langle 10 \cos 135^\circ, 10 \sin 135^\circ \rangle = \langle -5\sqrt{2}, 5\sqrt{2} \rangle$$

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$$\vec{F} = \langle 6\sqrt{3} - 5\sqrt{2}, 6 + 5\sqrt{2} \rangle = (6\sqrt{3} - 5\sqrt{2})\vec{i} + (6 + 5\sqrt{2})\vec{j}$$

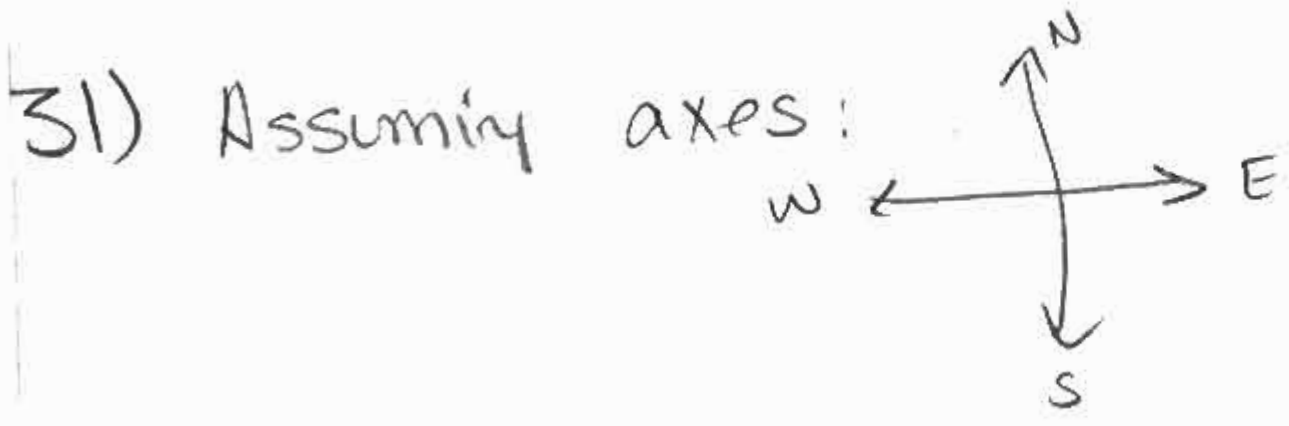
29 cont'd)

So  $|F| = \text{strength of resultant force}$   
 $= \sqrt{(6\sqrt{3} - 5\sqrt{2})^2 + (6 + 5\sqrt{2})^2}$   
 $\approx 13.5$

end

$$\tan \theta = \frac{y}{x} = \frac{6 + 5\sqrt{2}}{6\sqrt{3} - 5\sqrt{2}}$$

so  $\theta = \arctan\left(\frac{6 + 5\sqrt{2}}{6\sqrt{3} - 5\sqrt{2}}\right) \approx 76^\circ$

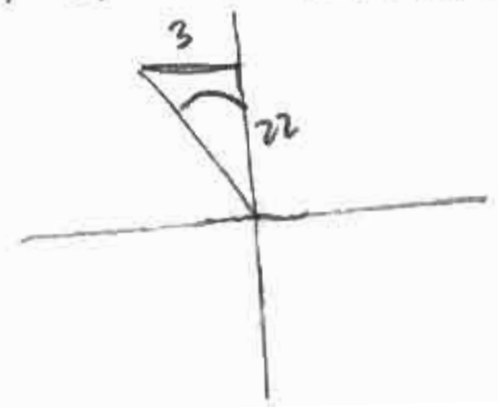


The velocity vector of the boat relative to the water is  $\langle 0, 22 \rangle$ . The velocity vector of the woman relative to the boat is  $\langle -3, 0 \rangle$ . So the velocity of the woman relative to the water is  $\langle 0, 22 \rangle + \langle -3, 0 \rangle = \langle -3, 22 \rangle$ .

Her speed is  $|\text{velocity}| = \sqrt{(-3)^2 + (22)^2} \approx 22.2 \text{ mi/hr}$

Her direction is  $N \theta^\circ W$  where

$$\tan \theta = \frac{3}{22} \quad \text{so } \theta \approx 8$$



so her direction is  $N 8^\circ W$