

53. Note that here the variables are  $m$  and  $b$ , and  $f(m, b) = \sum_{i=1}^n [y_i - (mx_i + b)]^2$ . Then

$$f_m = \sum_{i=1}^n -2x_i[y_i - (mx_i + b)] = 0 \text{ implies } \sum_{i=1}^n (x_i y_i - mx_i^2 - bx_i) = 0 \text{ or } \sum_{i=1}^n x_i y_i = m \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i$$

$$\text{and } f_b = \sum_{i=1}^n -2[y_i - (mx_i + b)] = 0 \text{ implies } \sum_{i=1}^n y_i = m \sum_{i=1}^n x_i + \sum_{i=1}^n b = m \left( \sum_{i=1}^n x_i \right) + nb. \text{ Thus we have}$$

the two desired equations. Now  $f_{mm} = \sum_{i=1}^n -2x_i^2$ ,  $f_{bb} = \sum_{i=1}^n -2 = -2n$  and  $f_{mb} = \sum_{i=1}^n -2x_i$ . And  $f_{mm}(m, b) > 0$

$$\text{always and } D(m, b) = 4n \left( \sum_{i=1}^n x_i^2 \right) - 4 \left( \sum_{i=1}^n x_i \right)^2 = 4 \left[ n \left( \sum_{i=1}^n x_i^2 \right) - \left( \sum_{i=1}^n x_i \right)^2 \right] > 0 \text{ always so the}$$

solutions of these two equations do indeed minimize  $\sum_{i=1}^n d_i^2$ .