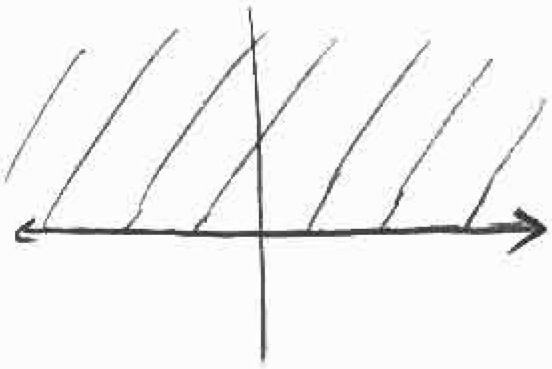
HW #6

Selected solutions

14.2 29) Determine where $f(x,y) = \arctan(x+\sqrt{y})$ is continuous.

$\arctan(u)$ is continuous everywhere on its domain. $\sqrt{y}$ is continuous for $y \geq 0$, so



$\leftarrow$ Domain is set $y \geq 0$

14.3 3) a) Estimate $f_T(-15,30)$ and $f_v(-15,30)$. What are the practical interpretations of these values?

$$f_T(-15,30) = \lim_{T \to -15} \frac{f(T,30) - f(-15,30)}{T-(-15)}$$

So, est, average the values of

$$\frac{f(-10,30) - f(-15,30)}{-10-(-15)} = \frac{-20-(-26)}{5} = \frac{6}{5}$$

$$\frac{f(-20,30) - f(-15,30)}{-20-(-15)} = \frac{-33-(-26)}{-5} = \frac{7}{5}$$

to get $f_T(-15,30) \approx \frac{13}{10}$. This is the rate at which the perceived temperature <u>increases</u> as the actual temperature <u>increases</u> if the wind speed is 30 km/h and the temperature starts at $-15°C$. The fact that this number, $\frac{13}{10} > 0$ means that perceived temperature does increase as actual temperature increases

3a cont'd)

Similarly, approximate $f_v(-15,30)$ by

$$\frac{f(-15,20)-f(-15,30)}{20-30} = \frac{-24-(-26)}{20-30} = \frac{2}{10} = \frac{1}{5}$$

$$\frac{f(-15,40)-f(-15,30)}{40-30} = \frac{-27-(-26)}{40-30} = \frac{-1}{10}$$

So $f_v(-15,30) \approx \frac{-3}{20}$. This is the rate at which the perceived temperature increases. as the wind speed increases from 30km/hr and the actual temperature is $-15°C$. The fact that this "increase", $\frac{-3}{20} < 0$ means in fact the perceived temperature decreases as wind speed increases.

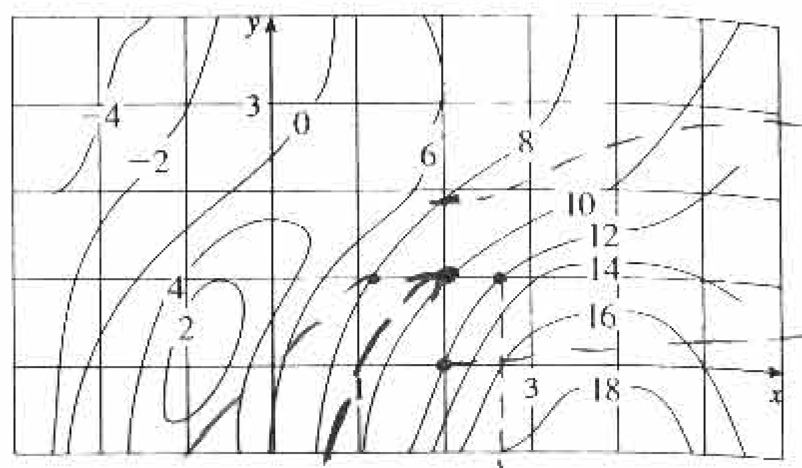
b) In general we expect $f_T(T,v) > 0$ since perceived temperature should increase if actual temprature does and we expect $f_v(T,v) < 0$ Since perceived temperature decreases as wind speed increases.

c) $\frac{\partial W}{\partial v}$ as $v \to \infty$ seems to go to 0

Since the wind chill increases slower and slower as wind speed increases.

8)

8. A contour map is given for a function f. Use it to estimate $f_x(2, 1)$ and $f_y(2, 1)$.



about $y = 1.9$

about $y = 0$

about $x = 1.2$

$(2,1)$

about $x = 2.6$

$(2,1)$ lies just about on the 10 contour, so $f(2,1) \approx 10$. To estimate $f_x(2,1)$, look at where the adjacent two contour lines cross $y=1$. Use these values to get two secant slopes around $(2,1)$ and average them to approximate $f_x(2,1)$:

$f(1.2,1) \approx 8 \qquad f(2.6,1) \approx 12$

$$\frac{f(1.2,1) - f(2,1)}{1.2 - 2} \approx \frac{8-10}{-.8} = \frac{20}{8} = \frac{5}{2}$$

$$\frac{f(2.6,1) - f(2,1)}{2.6 - 2} \approx \frac{12-10}{.6} = \frac{2}{.6} = \frac{10}{3}$$

$$f_x(2,1) \approx \left(\frac{5}{2} + \frac{10}{3}\right) \cdot \frac{1}{2} = \frac{35}{12}$$

Similarly, now look at where these contour lines cross $x=2$: $f(2,1.9) \approx 8$, $f(2,0) \approx 12$ now get these secant slopes and average:

$$\frac{f(2,1.9) - f(2,1)}{1.9 - 1} = \frac{8-10}{.9} = \frac{2}{.9}$$

$$f(2,0) - f(2,1) = 12 - 10 = 2$$

$$f_y(2,1) \approx \frac{1}{2}\left(\frac{2}{.9} + 2\right) \approx \frac{38}{18} = \frac{19}{9}$$