Hw #5

Selected Solutions

12.6 25)

21–28 Match the equation with its graph (labeled I–VIII). Give reasons for your choices.

21. $x^2 + 4y^2 + 9z^2 = 1$

22. $9x^2 + 4y^2 + z^2 = 1$

23. $x^2 - y^2 + z^2 = 1$

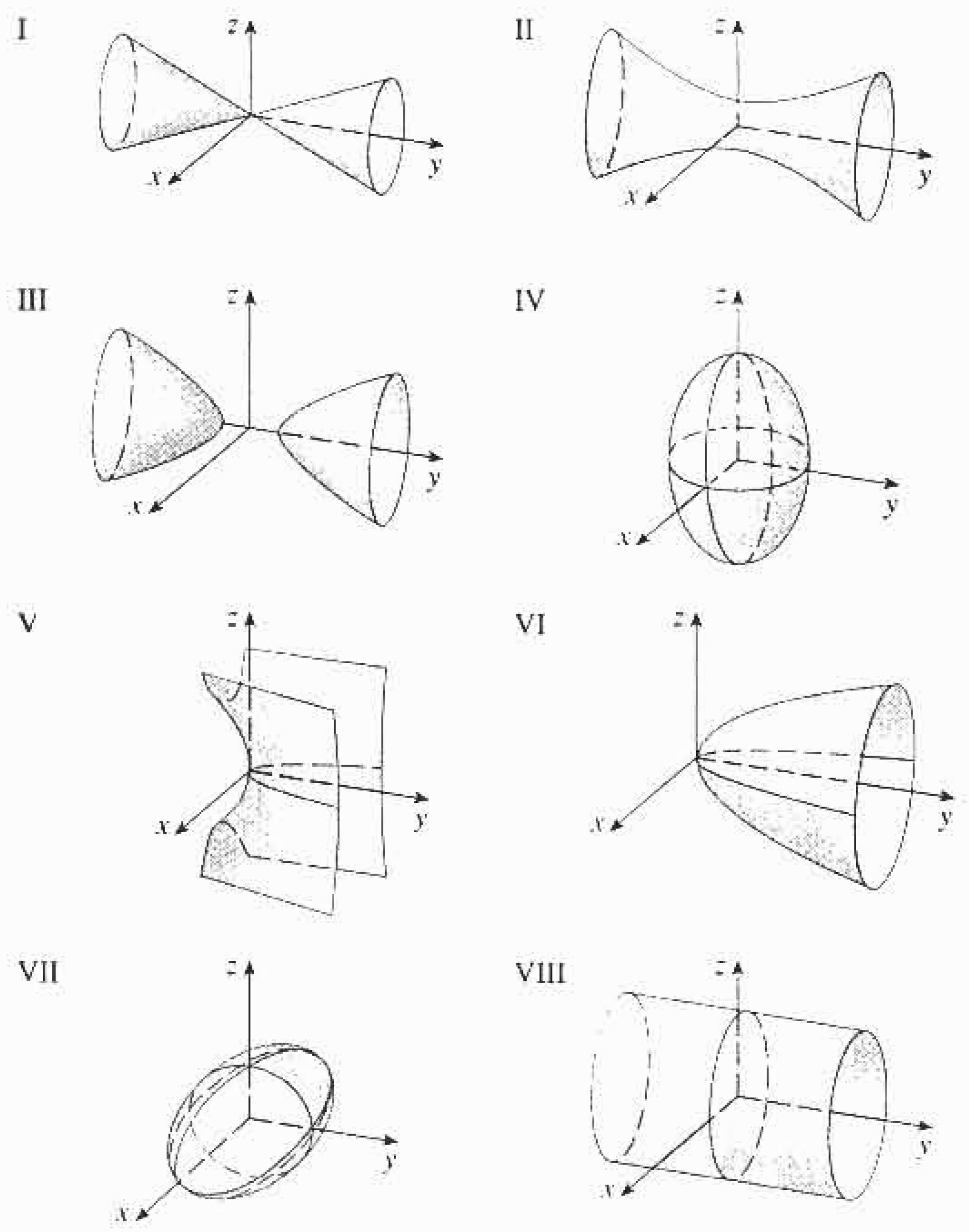
24. $-x^2 + y^2 - z^2 = 1$

25. $y = 2x^2 + z^2$

26. $y^2 = x^2 + 2z^2$

27. $x^2 + 2z^2 = 1$

28. $y = x^2 - z^2$



Note that there are no values of x, z that can make $y < 0$. So the graph must lie entirely to the right of the x-z plane (where $y=0$). This narrows our choices down to V and VI.

So consider slices in planes ∥ to xz plane, eg $y=1$.

$1 = 2x^2 + z^2$ is an ellipse, not a hyperbola, so 25 must have graph VI.

13.4

19) The position of a particle is given by $r(t) = \langle t^2, 5t, t^2 - 16t \rangle$. When is the speed a minimum? ______

We need to find the speed, $s(t) = |r'(t)|$ and then minimize it in the usual way setting $s'(t) = 0$ and solving.

$$s(t) = |r'(t)| = |\langle 2t, 5, 2t - 16 \rangle|$$

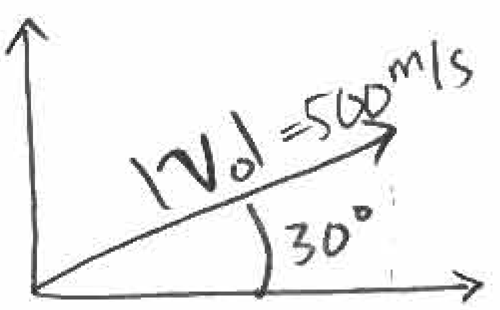
$$= \sqrt{(2t)^2 + 5^2 + (2t-16)^2} = \sqrt{4t^2 + 25 + 4t^2 - 32t + 256}$$

$$= \sqrt{8t^2 - 32t + 281}$$

$$s'(t) = \frac{1}{2}(8t^2 - 32t + 281)^{-\frac{1}{2}} \cdot (16t - 32) = 0$$

when the numerator, $16t - 32 = 0$, at $t = 2$.
If $t < 2$, eg $t = 0$, $s'(0) < 0$ and if $t > 2$, eg $t = 3$ then $s'(3) > 0$ so $t = 2$ does in fact give a minimum by the 1st derivative test.

23) A projectile is fired with initial speed 500 m/s and an angle of elevation 30°. Find a) range of projectile b) max height, c) speed at impact.



$$\vec{V}_0 = \langle 500\cos 30°, 500\sin 30° \rangle$$
$$= \langle 250\sqrt{3}, 250 \rangle$$

$$\vec{a} = \langle 0, -9.8 \text{ m/s}^2 \rangle$$

So integrating, $\vec{V} = \int_0^t \vec{a}(t)\,dt + V_0$

$\vec{v}(t) = \langle 250\sqrt{3},\ 250 - 9.8t \rangle$

$\vec{r}(t) = \int_0^t v(t)\,dt + r_0$

$= \langle 250\sqrt{3}t,\ 250t - 4.9t^2 \rangle$ is the position vector at time t. So now we can answer the questions.

a) Range is x value when $y = 0$.

$y(t) = 250t - 4.9t^2 = 0$ when $250 = 4.9t$, $t = \frac{250}{4.9} \approx 51$ sec

so $x(51) = 250\sqrt{3} \cdot 51 \approx 22{,}400$ meters

b) Max height is reached when $y' = 0$. This is when $y' = 250 - 9.8t = 0$, ie $t = \frac{250}{9.8} \approx 25.5$ sec.
The height then is
$y(25.5) \approx 3400$ meters

c) The speed at impact is $|\vec{v}(t_{impact})|$. We found $t_{impact} = 51$ sec in (a), so

$$|\vec{v}(51)| = \sqrt{(250\sqrt{3})^2 + (250 - 9.8 \cdot 51)^2} \approx 504 \text{ m/s}.$$

(note: we had some rounding errors. In fact, Speed on imact will be exactly speed fired if we discount air resistance and if height on impact = height initially).