QUIZ #3
Calculus 140
Fall 2005

Name KEY INSTRUCTOR

1) State and prove the sum rule for derivatives

If f and g are differentiable, then

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

proof

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \lim_{h\to 0} \frac{\left(f(x+h)+g(x+h)\right)-\left(f(x)+g(x)\right)}{h} \quad \text{by definition of } \frac{d}{dx}$$

$$= \lim_{h\to 0} \frac{\left(f(x+h)-f(x)\right)+\left(g(x+h)-g(x)\right)}{h} \quad \text{rearranging}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \quad \text{by limit sum rule}$$

$$= \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right] \quad \text{by definition of } \frac{d}{dx}. \ \blacksquare$$

2) If $y = \dfrac{2xe^x}{x^2+\sqrt{x}}$, find $\dfrac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{2xe^x}{x^2+\sqrt{x}}\right] = \frac{\frac{d}{dx}\left[2xe^x\right]\left(x^2+\sqrt{x}\right) - 2xe^x\frac{d}{dx}\left[x^2+\sqrt{x}\right]}{\left(x^2+\sqrt{x}\right)^2}$$

$$= \frac{2\frac{d}{dx}\left[xe^x\right]\left(x^2+\sqrt{x}\right) - 2xe^x\left(\frac{d}{dx}\left[x^2\right]+\frac{d}{dx}\left[x^{\frac{1}{2}}\right]\right)}{\left(x^2+\sqrt{x}\right)^2}$$

$$= \frac{2\left(\frac{d}{dx}[x]e^x + x\frac{d}{dx}[e^x]\right)\left(x^2+\sqrt{x}\right) - 2xe^x\left[2x+\frac{1}{2}x^{-\frac{1}{2}}\right]}{\left(x^2+\sqrt{x}\right)^2}$$

$$= \frac{2\left(1\cdot e^x + xe^x\right)\left(x^2+\sqrt{x}\right) - 2xe^x\left(2x+\frac{1}{2\sqrt{x}}\right)}{\left(x^2+\sqrt{x}\right)^2}$$

$$= \frac{2e^x\left[(1+x)\left(x^2+\sqrt{x}\right) - x\left(2x+\frac{1}{2\sqrt{x}}\right)\right]}{\left(x^2+\sqrt{x}\right)^2} = \frac{2e^x\left[x^3-x^2+x\sqrt{x}+\frac{1}{2}\sqrt{x}\right]}{\left(x^2+\sqrt{x}\right)^2}$$