Ex - 4.7

38). a) $E(v) = \dfrac{aLv^3}{v-u}$

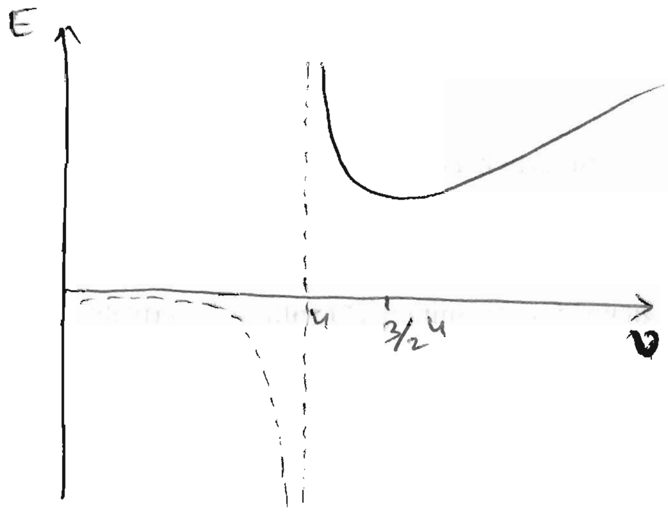
So,

$$E'(v) = aL\,\frac{(v-u)3v^2 - v^3}{(v-u)^2} = 0$$

when $2v^3 = 3uv^2 \Rightarrow 2v = 3u \Rightarrow v = \dfrac{3u}{2}$

The first derivative Test shows that this value of $v = \dfrac{3u}{2}$ gives the minimum value of E.

b)



Ex-4.10(74) $v'(t) = a(t) = -22$.

The initial velocity is $50\,mi/h = \dfrac{50 \cdot 5280}{3600} = \dfrac{220}{3}\,ft/s$

So, $v(t) = -22t + \dfrac{220}{3}$.

The car stops when $v(t) = 0 \Rightarrow t = \dfrac{220}{3 \cdot 22} = \dfrac{10}{3}$.

Since $s(t) = -11t^2 + \dfrac{220}{3}t$, the distance covered is

$$s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3}\cdot\frac{10}{3} = \frac{1100}{9} = 122.22\,ft$$

$\xi = (\frac{K_2}{K_1 A}) X \sim 2 * 10^{-10} M^{-1} [HBrO_2]$ $\eta = \frac{K_2}{K_3 A} Y \sim 3 * 10^6 M^{-1} [Br^-]$

$\rho = \frac{K_2 K_5}{2 K_1 K_3 A^2} Z \sim 6 * 10^3 M^{-1} [Ce^{+4}]$ $\tau = K_1 A \sim (0.1\, sec^{-1}) t$

Substituting those values in for (+), one can rewrite (+) as:

$$\epsilon \frac{d\xi}{d\tau} = \xi + \eta - \xi\eta - q\xi^2$$

$$\frac{d\eta}{d\tau} = 2h\rho - \eta - \xi\eta \qquad (*)$$

$$p \frac{d\rho}{d\tau} = \eta - \rho$$

where, $\epsilon = \frac{K_1}{K_2} \sim 2 * 10^{-4}$, $p = \frac{K_1 A}{K_5} \sim 300$, and $q = \frac{2 K_1 K_4}{K_2 K_3} \sim 8 * 10^{-6}$.

With dimensionless equations in hand, the three authors proceeded to analyze the steady state solutions. The most obvious solution would be the trivial case $\xi = \eta = \rho = 0$. The other two steady state solutions were obtained by examining (*) with each derivative set to 0 to get:

$$(a)\ \rho = \xi \qquad (b)\ \eta = \frac{2h\xi}{1+\xi} \qquad (c)\ q\xi^2 + (\eta - 1)\xi - \eta = 0$$

Combining (b) and (c) yields a quadratic equation for ξ. Using the quadratic formula

yields: $\xi = \frac{1 - 2h - q \pm \sqrt{(1-2h-q)^2 + 4q(2h+1)}}{2q}$. As $q \to 0$, by trichotomy, there are three possibilities for h: h < ½, h = ½, or h > ½. Noyes later suggested that the correct choice is that h ≈½ (Fowler) which corresponds to the point on the previous figure where p is smallest. By using h ≈½ and assuming that only the positive root of ξ has chemical significance, because there is no such thing as negative concentrations, they derived a steady state point, (ξ_0, η_0, ρ_0) where $\rho_0 = \xi_0 \approx \sqrt{2/q}$ and $\eta_0 \approx h + 1/2$. They then checked for stability at (ξ_0, η_0, ρ_0) by linearizing (*)