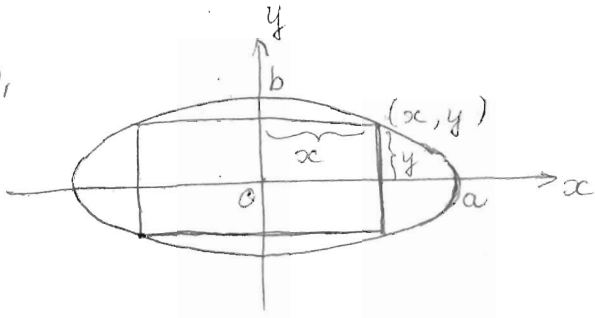


4.7 - Solution

20,



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Area of the rectangle:

$$A = (2x)(2y) = 4xy$$

We have to find A_{max} .

Since $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$$

$$y = \sqrt{\frac{b^2}{a^2}(a^2 - x^2)} = \frac{b}{a} \sqrt{a^2 - x^2}$$

So we maximize: $A(x) = 4xy = 4x \frac{b}{a} \sqrt{a^2 - x^2} = \frac{4b}{a} x (a^2 - x^2)^{1/2}$

Then: $A'(x) = \frac{4b}{a} \left[x \cdot \frac{1}{2} (a^2 - x^2)^{-1/2} (-2x) + (a^2 - x^2)^{1/2} \cdot 1 \right]$

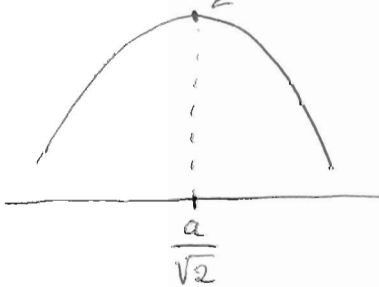
$$= \frac{4b}{a} \left[(a^2 - x^2)^{-1/2} (-x^2) + (a^2 - x^2)^{1/2} \right]$$

$$= \frac{4b}{a} (a^2 - x^2)^{-1/2} (-x^2 + a^2 - x^2)$$

$$= \frac{4b}{a} \cdot \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}}$$

$A'(x) = 0$
 $A(x) \text{ max}$

when $a^2 - 2x^2 = 0 \Rightarrow 2x^2 = a^2 \Rightarrow x = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$



x	$a/\sqrt{2}$		
$a^2 - 2x^2$	+	0	-
$A'(x)$	+	0	-
$A(x)$	increasing		decreasing

$\Rightarrow A(x) \text{ max when } x = \frac{a}{\sqrt{2}}$

$$y = \frac{b}{a} \sqrt{a^2 - x^2} = \frac{b}{a} \sqrt{a^2 - \frac{a^2}{2}}$$

$$= \frac{b}{a} \sqrt{\frac{a^2}{2}} = \frac{b}{\sqrt{2}}$$

Maximum area:

$$A(x) = 4 \left(\frac{a}{\sqrt{2}} \right) \left(\frac{b}{\sqrt{2}} \right) = 2ab.$$

$$38, a, E(v) = av^3 \cdot \frac{L}{v-u} = \frac{aLv^3}{v-u}$$

$$E'(v) = aL \cdot \frac{(v-u)3v^2 - v^3}{(v-u)^2} = 0 \quad (\text{minimize } E(v))$$

$$\frac{aLv^2[(3v-3u)-v]}{(v-u)^2} = 0$$

$$\frac{aLv^2(2v-3u)}{(v-u)^2} = 0$$

$$\text{Since } v \neq 0 \Rightarrow 2v-3u=0 \Rightarrow v = \frac{3}{2}u.$$

v	$\frac{3}{2}u$
$2v-3u$	- 0 +
$E'(v)$	- 0 +
$E(v)$	decreases increases

$\Rightarrow E(v)$ is minimized when $v = \frac{3}{2}u$

(or $v = \frac{3}{2}u$ gives the minimum value of E).

b) Graph of E :

