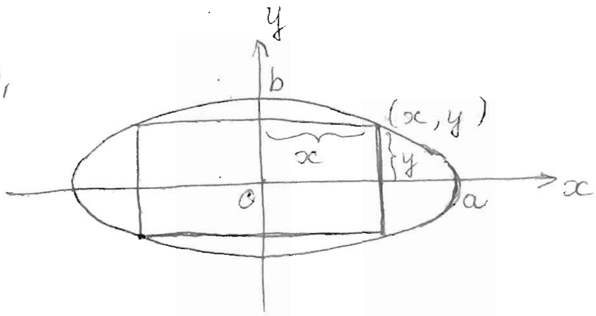
CALC HW # 7 Selected Solutions

4.7 – Solution

20,



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Area of the rectangle:

$$A = (2x)(2y) = 4xy$$

We have to find A max.

Since $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$$

$$y = \sqrt{\frac{b^2}{a^2}(a^2 - x^2)} = \frac{b}{a}\sqrt{a^2 - x^2}$$

So we maximize: $A(x) = 4xy = 4x\frac{b}{a}\sqrt{a^2 - x^2} = \frac{4b}{a}x(a^2 - x^2)^{1/2}$

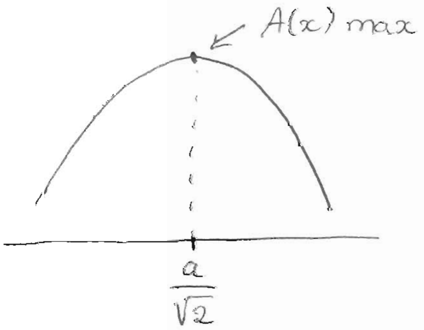
Then: $A'(x) = \frac{4b}{a}\left[x \cdot \frac{1}{2}(a^2 - x^2)^{-1/2}(-2x) + (a^2 - x^2)^{1/2} \cdot 1\right]$

$$= \frac{4b}{a}\left[(a^2 - x^2)^{-1/2}(-x^2) + (a^2 - x^2)^{1/2}\right]$$

$$= \frac{4b}{a}(a^2 - x^2)^{-1/2}(-x^2 + a^2 - x^2)$$

$$= \frac{4b}{a} \cdot \frac{a^2 - 2x^2}{\sqrt{a^2 - x^2}}$$

$A'(x) = 0$ when $a^2 - 2x^2 = 0 \Rightarrow 2x^2 = a^2 \Rightarrow x = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$



x		$a/\sqrt{2}$	
$a^2 - 2x^2$	$+$	0	$-$
$A'(x)$	$+$	0	$-$
$A(x)$	increasing		decreasing

$\Rightarrow A(x)$ max when $x = \frac{a}{\sqrt{2}}$

$$y = \frac{b}{a}\sqrt{a^2 - x^2} = \frac{b}{a}\sqrt{a^2 - \frac{a^2}{2}}$$

$$= \frac{b}{a}\sqrt{\frac{a^2}{2}} = \frac{b}{\sqrt{2}}$$

Maximum area:

$$A(x) = 4\left(\frac{a}{\sqrt{2}}\right)\left(\frac{b}{\sqrt{2}}\right) = 2ab.$$

38, a, $E(v) = av^3 \cdot \dfrac{L}{v-u} = \dfrac{aLv^3}{v-u}$

$$E'(v) = aL \cdot \frac{(v-u)3v^2 - v^3}{(v-u)^2} = 0 \quad (\text{minimize } E(v))$$

$$\frac{aLv^2[(3v-3u)-v]}{(v-u)^2} = 0$$

$$\frac{aLv^2(2v-3u)}{(v-u)^2} = 0$$

Since $v \neq 0 \Rightarrow 2v-3u = 0 \Rightarrow v = \frac{3}{2}u$.

v		$\frac{3}{2}u$	
$2v-3u$	$-$	0	$+$
$E'(v)$	$-$	0	$+$
$E(v)$	decreases		increases

$\Rightarrow$ $E(v)$ is minimized when $v = \frac{3}{2}u$

(or $v = \frac{3}{2}u$ gives the minimum value of E).

b) Graph of E: