calc HW #6 Selected Solutions

3.6:50) $\frac{d}{d\theta}[\arctan(\cos\theta)] = \frac{d}{d\theta}[\arctan u]$

where $u = \cos\theta$, and

$= \frac{dy}{d\theta}$, where $y = \arctan u$.

$= \frac{dy}{du}\frac{dy}{d\theta} = \frac{1}{1+u^2} \cdot -\sin\theta = \frac{1}{1+\cos^2\theta} \cdot -\sin\theta$

$= \frac{-\sin\theta}{1+\cos^2\theta}$

53) $\frac{d}{dx}[\arccos x]$:

$\arccos[\cos u] = u$ by def of arccos, so

Implicitly differentiating, we get:

$\frac{d}{du}[\arccos(\cos u)] = \frac{d}{du}[u] = 1$. The RHS is

RHS $= \frac{d}{du}[\arccos(x)]$ where $x = \cos u$ so $u = \arccos x$

$= \frac{dy}{du}$ where $y = \arccos x$, and by chain rule

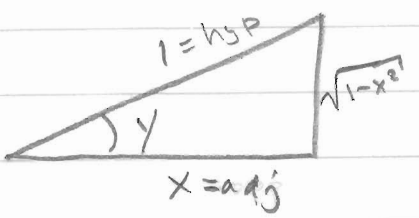
$\frac{dy}{du} = \frac{dy}{dx}\frac{dx}{du} = \underbrace{\frac{d}{dx}[\arccos x]}_{\text{what we want}} \cdot \frac{d}{du}[\cos u] = 1$

Thus:

$\frac{d}{dx}[\arccos x] \cdot -\sin u = 1$

$\frac{d}{dx}[\arccos x] = \frac{-1}{\sin u} = \frac{-1}{\sin[\arccos x]}$

Now simplify: If $y = \arccos x$ then $\cos y = x = \frac{x}{1}$ $= \frac{adj}{hyp}$



So other side is $\sqrt{1-x^2}$

Now $\sin[\arccos x] = \sin y = \frac{opp}{hyp} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

So $\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$

3.8:14) $\frac{d}{dy}\left[y \ln(1+e^y)\right]$

$= \frac{dy}{dy} \cdot \ln(1+e^y) + y \frac{d}{dy}\left(\ln(1+e^y)\right)$

$= 1 \cdot \ln(1+e^y) + y \frac{1}{(1+e^y)} \cdot \frac{d}{dy}\left[(1+e^y)\right]$ by formula 3 p. 244 (chain rule + ln rule)

$= \ln(1+e^y) + \frac{y(e^y)}{(1+e^y)}$.

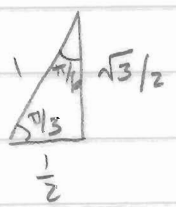
4.1:58) Find max & min for $f(x) = x - 2\cos x$ on $[-\pi, \pi]$.

Use closed interval method:

1) find critical points: $f'(x) = 1 + 2\sin x$

- This is always defined
- $1 + 2\sin x = 0$ when $\sin x = \frac{-1}{2}$

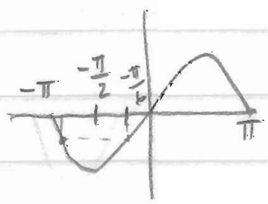
Draw a triangle with $\sin x = -\frac{1}{2}$:



$\sin(\pi/6) = \frac{1}{2}$ so

$\sin(-\pi/6) = -1/2$ since $\sin x$ is odd

There is another solution, too on $[-\pi, \pi]$:

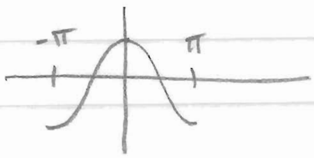


$-\pi + -\frac{\pi}{6}$ is the other solution $= -\frac{5\pi}{6}$

So the critical points are $-\pi, -\frac{5\pi}{6}, \frac{-\pi}{6}, \pi$.

evaluate: $f(x) = x - 2\cos x$

$f(-\pi) = -\pi - 2\cos(-\pi) = -\pi - 2 \cdot -1 = 2 - \pi \approx -1.14$



$f(\pi) = \pi - 2\cos\pi = \pi - 2(-1) = 2 + \pi$

≈ 5.14

$f(-\frac{5\pi}{6}) = -\frac{5\pi}{6} - 2\cos -\frac{5\pi}{6}$

$= \frac{-5\pi}{6} - 2\cos\frac{5\pi}{6}$ Since cos is even

$= \frac{-5\pi}{6} - 2 \cdot \frac{\sqrt{3}}{2} = \frac{-5\pi}{6} - \sqrt{3} \approx -4$

$f(\frac{-\pi}{6}) = \frac{-\pi}{6} - 2\cos\frac{-\pi}{6} = \frac{-\pi}{6} - 2\cos\frac{\pi}{6}$

$= \frac{-\pi}{6} - 2\frac{\sqrt{3}}{2} = \frac{-\pi}{6} - \sqrt{3} \approx 2$

The min is $f(\frac{-5\pi}{6}) = \frac{-5\pi}{6} - \sqrt{3}$

The max is $f(\pi) = 2 + \pi$.