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Ex-2.1

3) The point $P(1, \frac{1}{2})$ lies on the curve $y = x/(1+x)$

a) $Q(x, x/(1+x))$

slope of PQ, $m = \left(\frac{x}{1+x} - \frac{1}{2}\right) \Big/ \left(x-1\right)$

$$= \frac{2x-1-x}{2(1+x)} \div (x-1)$$

$$= \frac{x-1}{2(1+x)} \cdot \frac{1}{(x-1)}$$

$$= \frac{(x-1)}{2(x^2-1)} = \frac{x-1}{2x^2-2}$$

i) $x = 0.5$

$$m = \frac{0.5-1}{2(0.5)^2-2} = \frac{-0.5}{-1.5} = 0.333333$$

ii) $x = 0.9$

$$m = \frac{0.9-1}{2x^2-2} = \frac{0.9-1}{2(0.9)^2-2} = 0.263158$$

iii) $x = 0.99$

$$m = \frac{0.99-1}{2(0.99)^2-2} = 0.251256$$

iv) 0.999

$$m = \frac{0.999-1}{2(0.999)^2-2} = 0.250125$$

v) 1.5

$$m = \frac{1.5-1}{2(1.5)^2-2} = 0.2$$

vi) $x = 1.1$

$$m = \frac{1.1 - 1}{2(1.1)^2 - 2} = 0.238095$$

vii) $x = 1.01$

$$m = \frac{1.01 - 1}{2(1.01)^2 - 2} = 0.248756$$

viii) $x = 1.001$

$$m = \frac{1.001 - 1}{2(1.001)^2 - 2} = 0.249875$$

6. Initial velocity of an arrow, $V_i = 58 m/s$

$h = 58t - 0.83t^2$

a) Find the average velocity over the given time interval.

i) $t = [1, 2]$

$h_1 = 58 - 0.83$, $h_2 = 58(2) - 0.83(2)^2$, $\Delta t = 1$.

$V_{avg} = 55.51 m/s$

ii) $[1, 1.5]$

$$V_{avg} = \frac{58(1.5) - 0.83(1.5)^2 - 58 + 0.83}{0.5} = 55.925 m/s$$

iii) $t = [1, 1.1]$

$$V_{avg} = \frac{58(1.1) - 0.83(1.1)^2 - 58 + 0.83}{0.1} = 56.257 m/s$$

iv) $t = [1, 1.01]$

$$V_{avg} = \frac{58(1.01) - 0.83(1.01)^2 - 58 + 0.83}{0.01} = 56.3317 m/s$$

v) $t = [1, 1.001]$

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$$V_{avg} = \frac{58(1.001) - 0.83(1.001)^2 - 58 + 0.83}{0.001}$$

(v) $\therefore V_{avg} = 56.33917$ m/s 4.

b) Instantaneos velocity after one second,

$$V = \left.\frac{dh}{dt}\right|_{t=1} = 58 - (2 \times 0.83)t$$

$$= 58 - (2 \times 0.83) = 56.34 \text{ m/s}$$

$\therefore V = 56.34$ m/s, instantaneous velocity after one second.

④ The point $P(2, \ln 2)$ lies on the curve $y = \ln x$

a) $Q(x, \ln x)$

Slope of $PQ = \frac{\ln x - \ln 2}{x - 2}$

i) when $x = 1.5$

$$m = \frac{\ln 1.5 - \ln 2}{1.5 - 2} = \boxed{0.575364}$$

ii) when $x = 1.9$

$m = \boxed{0.512933}$

iii) when $x = 1.99$

$m = \boxed{0.501254}$

iv) when $x = 1.999$

$m = \boxed{0.500125}$

v) when $x = 2.5$

$\boxed{m = 0.446287}$

vi) when $x = 2.1$

$\boxed{m = 0.487902}$

vii) $x = 2.01$

$$\boxed{m = 0.498754}$$

viii) 2.001

$$\boxed{m = 0.499875}$$

b)

$$y = \ln x$$

$$\frac{dy}{dx} = \frac{d}{dx}\ln x = \frac{1}{x}$$

So,

$$\left.\frac{dy}{dx}\right|_{\substack{y=\ln 2 \\ x=2}} = \frac{1}{x} = \frac{1}{2}$$

$\therefore$ The slope of the tangent line to the curve at $P(2, \ln 2) = \frac{1}{2}$.

c)

$$m = \frac{y - \ln 2}{x - 2}$$

or, $\frac{1}{2}(x-2) = y - \ln 2$

or, $\frac{1}{2}x - 1 = y - \ln 2$

or, $y = \frac{1}{2}x + \ln 2 - 1$

$\therefore$ $y = \frac{1}{2}x + \ln 2 - 1$, is the required eq^n of tangent

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d) $y = \ln x$