Calc 140 HW #4
SELECTED SOLUTIONS

Ex - 3.1

58) If $x \ge 1$, then $h(x) = |x-1| + |x+2| = x-1+x+2 = 2x+1$

If $-2 < x < 1$, then $h(x) = -(x-1) + x+2 = 3$

If $x \le -2$, then $h(x) = -(x-1) - (x+2) = -2x-1$. Therefore

$$h(x) = \begin{cases} -2x-1 & \text{if } x \le -2 \\ 3 & \text{if } -2 < x < 1 \\ 2x+1 & \text{if } x \ge 1 \end{cases}$$

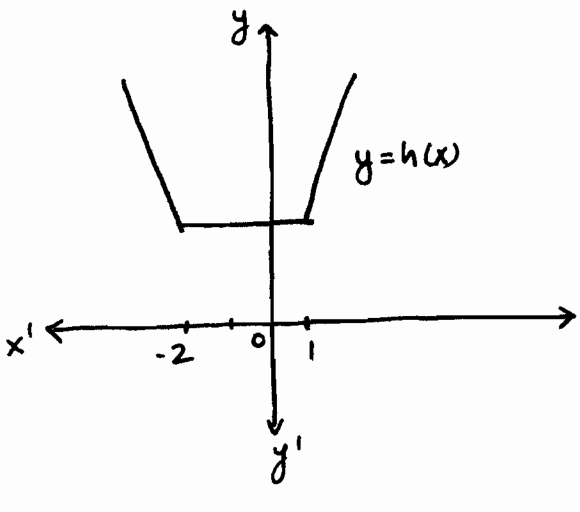
$$\Longrightarrow \quad h'(x) = \begin{cases} -2 & \text{if } x < -2 \\ 0 & \text{if } -2 < x < 1 \\ 2 & \text{if } x > 1 \end{cases}$$

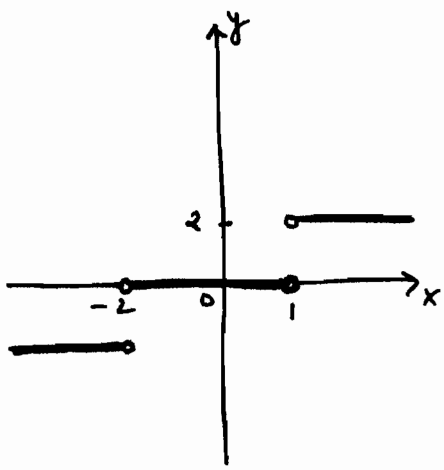
To see that $h'(1) = \lim\limits_{x \to 1} \dfrac{h(x)-h(1)}{x-1}$ does not exist, observe

that $\lim\limits_{x \to 1^-} \dfrac{h(x)-h(1)}{x-1} = \lim\limits_{x \to 1^-} \dfrac{3-3}{3-1} = 0$ but

$\lim\limits_{x \to 1^+} \dfrac{h(x)-h(1)}{x-1} = \lim\limits_{x \to 1^+} \dfrac{2x-2}{x-1} = 2$. i.e $h'(1)$ doesn't exist.

Similarly you can prove $h'(-2)$ doesn't exist.





Ex - 3.3

25) a) $[C] = \dfrac{a^2kt}{akt+1}$

rate of reaction $= \dfrac{d[C]}{dt} = \dfrac{d}{dt}\left[\dfrac{a^2kt}{akt+1}\right]$

$$= \frac{(akt+1)\frac{d}{dt}[a^2kt] - (a^2kt)\frac{d}{dt}[akt+1]}{(akt+1)^2}$$

$$= \frac{(akt+1)(a^2k) - (a^2kt)(ak)}{(akt+1)^2}$$

$$= \frac{a^2k[akt+1-akt]}{(akt+1)^2}$$

$$\therefore \frac{d[C]}{dt} = \frac{a^2k}{(akt+1)^2}$$

b) If $x = [C]$, then $a - x = a - \dfrac{a^2kt}{akt+1} = \dfrac{a^2kt + a - a^2kt}{akt+1}$

$$= \frac{a}{akt+1}$$

So, $k(a-x)^2 = k\left(\dfrac{a}{akt+1}\right)^2 = \dfrac{a^2k}{(akt+1)^2} = \dfrac{d[C]}{dt}$

$\therefore \dfrac{d[C]}{dt} = \dfrac{dx}{dt} = k(a-x)^2$ [From part a].

c) As $t \to \infty$, $[C] = \dfrac{a^2kt}{akt+1}$

$$\lim_{t\to\infty} \frac{a^2kt}{akt+1} = \lim_{t\to\infty}\left[\frac{a^2kt}{t} \div \frac{akt+1}{t}\right] = \lim_{t\to\infty}\left[\frac{a^2k}{ak+\underbrace{(1/t)}_{\to 0}}\right]$$

$$= \frac{a^2k}{ak} = a \text{ moles}/L.$$

d) As $t \to \infty$, $\dfrac{d[C]}{dt} = \dfrac{a^2 k}{(ak+1)^2} \Rightarrow 0$

$$\therefore \lim_{t \to \infty} \frac{d[C]}{dt} = 0$$

e) As t increases, nearly all of the reactants A and B are converted into product C. In practical terms, the reaction virtually stops.

Solution

3.3

28 $$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

a, i, $$f = \underbrace{\frac{1}{2}\sqrt{\frac{T}{\rho}}}_{\text{constant}} \cdot L^{-1}$$

$$\Rightarrow \frac{df}{dL} = \frac{1}{2}\sqrt{\frac{T}{\rho}}\,(-1)\,L^{-2} = -\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}$$

ii, $$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}} = \underbrace{\frac{1}{2L\sqrt{\rho}}}_{\text{constant}} \cdot T^{1/2}$$

$$\Rightarrow \frac{df}{dT} = \frac{1}{2L\sqrt{\rho}} \cdot \frac{1}{2} \cdot T^{-1/2} = \frac{1}{4L\sqrt{\rho}\cdot\sqrt{T}} = \frac{1}{4L\sqrt{T\rho}}$$

iii, $$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}} = \underbrace{\left(\frac{\sqrt{T}}{2L}\right)}_{\text{constant}} \cdot \rho^{-1/2}$$

$$\frac{df}{d\rho} = \frac{\sqrt{T}}{2L}\left(-\frac{1}{2}\right)\rho^{-3/2} = -\frac{\sqrt{T}}{4L\rho^{3/2}}$$

b, i, $$\frac{df}{dL} = -\frac{1}{2L^2}\sqrt{\frac{T}{\rho}} < 0$$

$\Rightarrow$ When $L \downarrow$, $f \uparrow$ $\Rightarrow$ the pitch ~~is~~ ~~ki~~ increases $\rightarrow$ higher note

ii, $$\frac{df}{dT} = \frac{1}{4L\sqrt{T\rho}} > 0$$

$\Rightarrow$ When $T \uparrow$, $f \uparrow$ $\rightarrow$ the pitch increases $\rightarrow$ higher note

iii, $$\frac{df}{d\rho} = -\frac{\sqrt{T}}{4L\rho^{3/2}} < 0$$

$\Rightarrow$ When $\rho \uparrow$, $f \downarrow$ $\Rightarrow$ the pitch decreases $\rightarrow$ lower note.

3.4

18. $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$

(Quotient Rule) $= \frac{\cos x\,(1)' - 1.(\cos x)'}{\cos^2 x}$

$= \frac{\cos x . 0 - 1(-\sin x)}{\cos^2 x}$

$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$= \sec x . \tan x$