

Calc HW #3 selected solutions

2.6.6b

a) If f is defined on an interval $(-\infty, a)$ then
 $\lim_{x \rightarrow -\infty} f(x) = -\infty$ if for all M there is

some N such that $f(x) < M$ when $x < N$.

b) $\lim_{x \rightarrow -\infty} 1+x^3$ $1+x^3 < M$ if $x^3 < M-1$
if $x < \sqrt[3]{M-1}$

So let $N = \sqrt[3]{M-1}$.

2.7.8

slope = $\lim_{x \rightarrow a} \frac{\sqrt{2x+1} - \sqrt{2a+1}}{x-a}$

= $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - \sqrt{9}}{x-4}$

= $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - \sqrt{9}}{x-4} \cdot \frac{\sqrt{2x+1} + \sqrt{9}}{\sqrt{2x+1} + \sqrt{9}}$

= $\lim_{x \rightarrow 4} \frac{2x+1-9}{(x-4)(\sqrt{2x+1} + 3)} = \lim_{x \rightarrow 4} \frac{2x-8}{(x-4)(\sqrt{2x+1} + 3)}$

= $2 \lim_{x \rightarrow 4} \frac{\cancel{x-4}}{(x-4)(\sqrt{2x+1} + 3)}$

= $2 \lim_{x \rightarrow 4} \frac{1}{\sqrt{2x+1} + 3} = 2 \cdot \frac{1}{\sqrt{2 \cdot 4 + 1} + 3} = \frac{2}{6} = \frac{1}{3}$

So the tangent line is

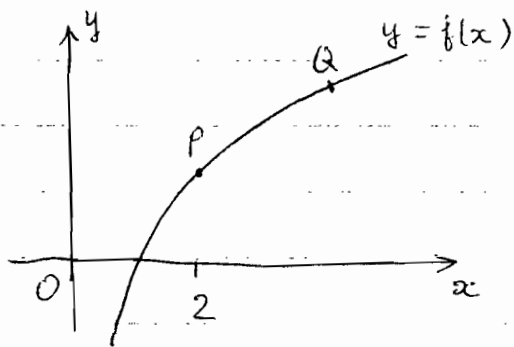
$y - y_0 = m(x - x_0)$

$y - 3 = \frac{1}{3}(x - 4)$ or $y = \frac{1}{3}x + \frac{5}{3}$.

2.8 Solution

$$2, \quad \frac{1}{2} [f(4) - f(2)] = \frac{f(4) - f(2)}{4-2}$$

$$f(3) - f(2) = \frac{f(3) - f(2)}{3-2}$$



When $x \rightarrow 2$, PQ becomes steeper.

\Rightarrow Its slope increases.

Therefore,

$$0 < \frac{f(4) - f(2)}{4-2} < \frac{f(3) - f(2)}{3-2} < \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x-2}$$

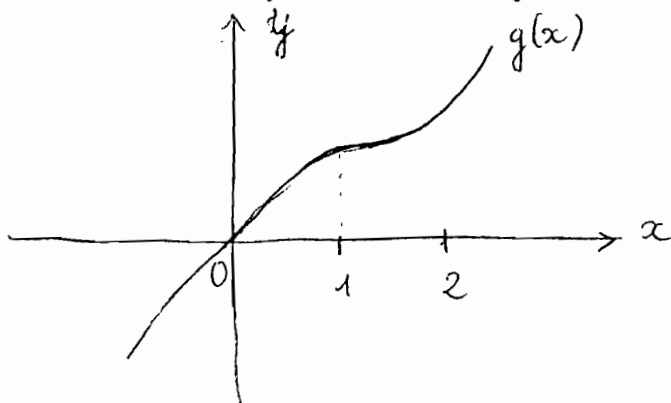
$$\text{OR, } 0 < \frac{1}{2} [f(4) - f(2)] < f(3) - f(2) < f'(2)$$

6,

$$g(0) = 0, \quad g'(0) = 3 > 0, \quad g'(1) = 0, \quad g'(2) = 1 > 0$$

$g(x)$ increasing

$g(x)$ increasing



$$2x - 2.8$$

16) Find $f'(a)$

$$f(x) = \frac{x^2 + 1}{x - 2}$$

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{(a+h)^2 + 1}{(a+h) - 2} - \frac{a^2 + 1}{a - 2} \right] \div [h]$$

$$= \lim_{h \rightarrow 0} \left[\frac{a^2 + 2ah + h^2 + 1}{(a+h) - 2} - \frac{a^2 + 1}{a - 2} \right] \div (h)$$

$$= \lim_{h \rightarrow 0} \left[\frac{(a-2)(a^2 + 2ah + h^2 + 1) - (a^2 + 1)(a+h-2)}{(a-2)(a+h-2)} \right] \cdot \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(a^3 - 2a^2 + 2a^2h - 4ah + ah^2 - 2h^2 + a - 2) - (a^3 + a^2h - 2a^2 + a - 2)}{h(a+h-2)(a-2)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{a^3} - \cancel{2a^2} + 2a^2h - 4ah + ah^2 - 2h^2 + \cancel{a} - \cancel{2} - \cancel{a^3} - a^2h + \cancel{2a^2} - \cancel{a} - \cancel{2}}{h(a+h-2)(a-2)}$$

$$= \lim_{h \rightarrow 0} \frac{(2a^2h - 4ah + ah^2 - 2h^2 - a^2h - h)}{h(a+h-2)(a-2)}$$

$$= \lim_{h \rightarrow 0} \frac{(2a^2 - 4a + ah - 2h - a^2 - 1)h}{(a+h-2)(a-2)h}$$

$$= \frac{(2a^2 - a^2 - 1) - 4a}{(a-2)(a-2)}$$

$$= \frac{a^2 - 4a - 1}{(a-2)^2}$$

$$\therefore f'(a) = \frac{a^2 - 4a - 1}{(a-2)^2}$$

2.8.28

a) If $f(t)$ = number of bacteria after t hours,
then $f'(t)$ = rate at which population
is growing, in bacteria/hr

b) If there are unlimited nutrients, then
the larger the population, the faster
it grows, since there are more bacteria
to reproduce. We expect the
population to just grow, so we expect
 $f'(5) < f'(10)$.

If resources are limited, it depends
how quickly they become exhausted.
It could go either way, depending
on if there were a lot of
resources, in which case $f'(5) < f'(10)$
or if they are very limited, in which
case the population might start to
starve & die off between times 5
and 10, then $f'(5) > f'(10)$