Calc HW #3 selected solutions

2.6.66

a) If f is defined on an interval $(-\infty, a)$ then $\lim_{x\to-\infty} f(x) = -\infty$ if for all M there is

2 some N such that $f(x) < M$ when $x < N$.

b) $\lim_{x\to-\infty} 1+x^3$ $\quad 1+x^3 < M$ if $x^3 < M-1$

2 if $x < \sqrt[3]{M-1}$

So let $N = \sqrt[3]{M-1}$.

2.7.8

$$\text{slope} = \lim_{x\to a} \frac{\sqrt{2x+1} - \sqrt{2a+1}}{x-a}$$

$$= \lim_{x\to 4} \frac{\sqrt{2x+1} - \sqrt{9}}{x-4}$$

$$= \lim_{x\to 4} \frac{\sqrt{2x+1} - \sqrt{9}}{x-4} \cdot \frac{\sqrt{2x+1} + \sqrt{9}}{\sqrt{2x+1} + \sqrt{9}}$$

2

$$= \lim_{x\to 4} \frac{2x+1-9}{(x-4)(\sqrt{2x+1}+3)} = \lim_{x\to 4} \frac{2x-8}{(x-4)(\sqrt{2x+1}+3)}$$

$$= 2\lim_{x\to 4} \frac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{2x+1}+3)}$$

$$= 2\lim_{x\to 4} \frac{1}{\sqrt{2x+1}+3} = 2 \cdot \frac{1}{\sqrt{2\cdot 4+1}+3} = \frac{2}{6} = \frac{1}{3}$$

So the tangent line is

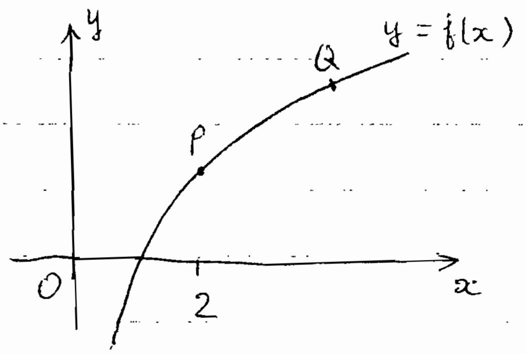
$$y - y_0 = m(x - x_0)$$

$$y - 3 = \tfrac{1}{3}(x-4) \quad \text{or} \quad y = \tfrac{1}{3}x + \tfrac{5}{3}.$$

2.8 Solution

2, $\frac{1}{2}[f(4)-f(2)] = \dfrac{f(4)-f(2)}{4-2}$

$f(3)-f(2) = \dfrac{f(3)-f(2)}{3-2}$



When $x \to 2$, PQ becomes steeper

$\Rightarrow$ Its slope increases,

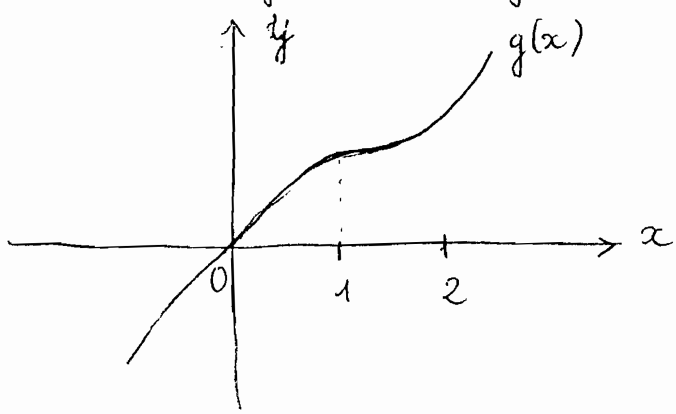
Therefore:

$$0 < \frac{f(4)-f(2)}{4-2} < \frac{f(3)-f(2)}{3-2} < \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$$

OR: $0 < \frac{1}{2}[f(4)-f(2)] < f(3)-f(2) < f'(2)$

6, $g(0)=0$, $\underset{\Downarrow}{g'(0) = 3 > 0}$, $g'(1) = 0$, $\underset{\Downarrow}{g'(2) = 1 > 0}$

g(x) increasing g(x) increasing



$\underline{Ex - 2.8}$

16) Find $f'(a)$

$$f(x) = \frac{x^2+1}{x-2}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \left[\frac{(a+h)^2+1}{(a+h)-2} - \frac{a^2+1}{a-2}\right] \div [h]$$

$$= \lim_{h\to 0} \left[\frac{a^2+2ah+h^2+1}{(a+h)-2} - \frac{a^2+1}{a-2}\right] \div (h)$$

$$= \lim_{h\to 0} \left[\frac{(a-2)(a^2+2ah+h^2+1) - (a^2+1)(a+h-2)}{(a-2)(a+h-2)}\right] \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{(a^3 - 2a^2 + 2a^2h - 4ah + ah^2 - 2h^2 + a - 2) - (a^3 + a^2h - 2a^2 + a + h -}{h(a+h-2)(a-2)}$$

$$= \lim_{h\to 0} \frac{(\cancel{a^3} - \cancel{2a^2} + 2a^2h - 4ah + ah^2 - 2h^2 + \cancel{a} - \cancel{2} - \cancel{a^3} - a^2h + \cancel{2a^2} - \cancel{a} - h}{h(a+h-2)(a-2)}$$

$$= \lim_{h\to 0} \frac{(2a^2h - 4ah + ah^2 - 2h^2 - a^2h - h)}{h(a+h-2)(a-2)}$$

$$= \lim_{h\to 0} \frac{(2a^2 - 4a + ah - 2h - a^2 - 1)\cancel{h}}{(a+h-2)(a-2)\cancel{h}}$$

$$= \frac{(2a^2 - a^2 - 1 - 4a)}{(a-2)(a-2)}$$

$$= \frac{a^2 - 4a - 1}{(a-2)^2}$$

$$\boxed{\therefore\ f'(a) = \frac{a^2 - 4a - 1}{(a-2)^2}}$$

2.8.28 a) If $f(t)$ = number of bacteria after t hours, then $f'(t)$ = rate at which population is growing, in bacteria/hr

b) If there are unlimited nutrients, then the larger the population, the faster it grows, since there are more bacteria to reproduce. We expect the population to just grow, so we expect $f'(5) < f'(10)$.

If resources are limited, it depends how quickly they become exhausted. It could go either way, depending on if there were a lot of resources, in which case $f'(5) < f'(10)$ or if they are very limited, in which case the population might start to starve & die off between times 5 and 10, then $f'(5) > f'(10)$