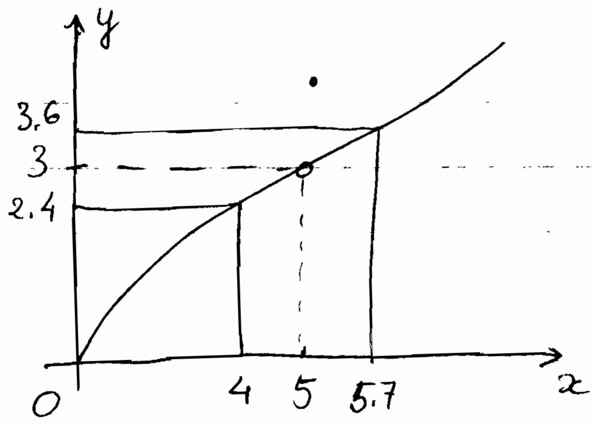
Selected Solutions to HW #2 Calc 140

– 2.4 –

4



Definition of limit

On the left side of $f(5)$, we need:

$$|x-5| < |4-5| \Rightarrow |x-5| < 1$$

On the right side of $f(5)$, we need:

$$|x-5| < 5.7-5 \Rightarrow |x-5| < 0.7$$

For both conditions to be satisfied at once, we need more restrictive condition to hold $\rightarrow |x-5| < 0.7$

$\Rightarrow \delta = 0.7$ or any smaller ~~possi~~ positive number.

14. a, To maintain temperature at 200°C :

$$T(w) = 200 = 0.1w^2 + 2.155w + 20$$

$$0.1w^2 + 2.155w - 180 = 0$$

$$w = \frac{-2.155 \pm \sqrt{2.155^2 - 4.(1).(-180)}}{2(.1)}$$

$w = 32.99 \approx 33$ or $w = -54.54$ (don't need negative value)

b, $T(w) = 199°C \Rightarrow 0.1w^2 + 2.155w + 20 = 199$

$$\Rightarrow w \approx 32.88$$

$T(w) = 201°C \Rightarrow 0.1w^2 + 2.155w + 20 = 201$

$\Rightarrow w \approx 33.12$

$\rightarrow$ Wattage is allowed for input power:

$32.88 < w < 33.12$

c, x is the input power

$f(x)$ is the temperature

a is the target input power given in part a,.

L is the target temperature, which is 200°C

ε is 1°C

δ is 0.12 watts

$(\delta \approx 33.12 - 33 \approx 33 - 32.88)$

Ex - 2.4

Amol

18). Prove the Statement using the ϵ, δ definition of limit and illustrate with diagram.

$$\lim_{x\to 4}(7-3x) = -5$$

Given: $\epsilon > 0$, we need $\delta > 0$ such that if $0 < |x-4| < \delta$ then $|(7-3x) - (-5)| < \epsilon$.

$$|(7-3x) - (-5)| < \epsilon$$
$$\Rightarrow |-3x+12| < \epsilon$$
$$\Rightarrow |-3|\,|x-4| < \epsilon$$
$$\Rightarrow |x-4| < \epsilon/3.$$

So, let say $\delta = \epsilon/3$ then $0 < |x-4| < \delta$ $\Rightarrow |(7-3x) - (-5)| < \epsilon$. Thus, $\lim_{x\to 4}(7-3x) = -5$ by the definition of limit.

Ex - 2.5

46) Use the intermediate Value theorem to prove that there is a positive number c such that $c^2=2$. (This proves the existence of the number $\sqrt{2}$).

$\rightarrow$ $f(x) = x^2$ is continuous on the interval $[1,2]$, $f(1)=1$, and $f(2)=4$. Since $f(1) \neq f(2)$, there exist a number c in $(1,2)$ such that $f(c) = c^2 = 2$; by the intermediate value theorem.

QUIZ #1
CALCULUS 140
Fall 2005, HUNSICKER

NAME KEY Honor Pledge

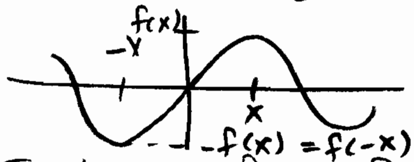
1) Define even and odd functions. Draw a picture or give an example to explain.

<u>Def</u> If $f(x)$ satisfies $f(x) = f(-x)$ for every number x in its domain then we say f is an <u>even</u> function. If $f(-x) = -f(x)$ for all x in its domain, we say f is an <u>odd</u> function.

<u>Idea</u> Even functions are symmetric about the y-axis
Odd functions are symmetric about the origin.

<u>Ex</u> $f(x) = \cos x$ is even:

<u>Ex</u> $f(x) = \sin x$ is odd.



2) Let $f(x) = (1 + \frac{\ln 2}{x})^x$. Evaluate $f(x)$ for $x = 1, .5, .1, .01$ and $x = -1, -.5, -.1, -.01$. Use this information to guess $\lim_{x \to 0} (1 + \frac{\ln 2}{x})^x$.

x	$(1 + \frac{\ln 2}{x})^x$
1	1.69
.5	1.54
.1	1.23
.01	1.09
-.01	undefined
-.1	undefined
-.5	undefined
-1	3.25889

Since f is undefined for x close to 0 but negative, $\lim_{x \to 0} (1 + \frac{\ln 2}{x})^x$ doesn't exist.

However, from the values for $x > 0$, we can guess the right-handed limit is

$$\lim_{x \to 0^+} (1 + \frac{\ln 2}{x})^x = 1.$$