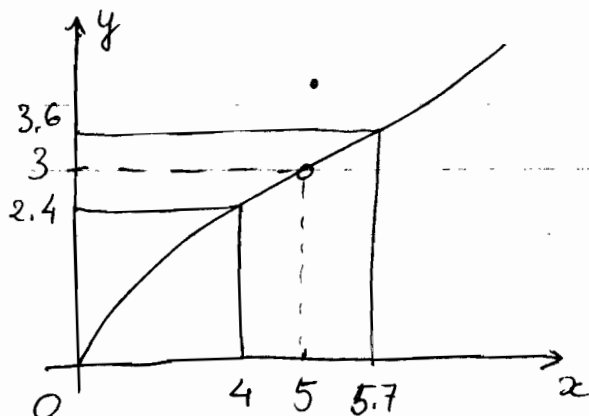


Selected Solutions to HW #2 Calc 140

- 2.4 -

4



Definition
of limit

{ On the left side of $f(5)$, we need:

$$|x-5| < |4-5| \Rightarrow |x-5| < 1$$

{ On the right side of $f(5)$, we need:

$$|x-5| < |5.7-5| \Rightarrow |x-5| < 0.7$$

For both conditions to be satisfied at once,
we need more restrictive condition to hold $\rightarrow |x-5| < 0.7$
 $\Rightarrow \delta = 0.7$ or any smaller ~~possi~~ positive number.

14.

a, To maintain temperature at 200°C :

$$T(w) = 200 = 0.1w^2 + 2.155w + 20$$

$$0.1w^2 + 2.155w - 180 = 0$$

$$w = \frac{-2.155 \pm \sqrt{2.155^2 - 4(0.1)(-180)}}{2(0.1)}$$

$$w = 32.99 \approx 33 \text{ or } w = -54.54 \text{ (don't need negative value)}$$

b, $T(w) = 199^\circ\text{C} \Rightarrow 0.1w^2 + 2.155w + 20 = 199$

$$\Rightarrow w \approx 32.88$$

$$T(w) = 201^\circ\text{C} \Rightarrow 0.1w^2 + 2.155w + 20 = 201$$

$$\Rightarrow w \approx 33.12$$

→ Wattage is allowed for input power:

$$32.88 < w < 33.12$$

c, x is the input power

$f(x)$ is the temperature

a is the target input power given in part a,

L is the target temperature, which is 200°C

ϵ is 1°C

δ is 0.12 watts

$$(\delta \approx 33.12 - 33 \approx 33 - 32.88)$$

Ex-2.4

Answer

18). Prove the statement using the ϵ, δ definition of limit and illustrate with diagram.

$$\lim_{x \rightarrow 4} (7 - 3x) = -5$$

Given: $\epsilon > 0$, we need $\delta > 0$ such that if $0 < |x - 4| < \delta$

$$\text{then } |(7 - 3x) - (-5)| < \epsilon.$$

$$|(7 - 3x) - (-5)| < \epsilon$$

$$\Rightarrow |-3x + 12| < \epsilon$$

$$\Rightarrow |-3||x - 4| < \epsilon$$

$$\Rightarrow |x - 4| < \epsilon/3.$$

So, let say $\delta = \epsilon/3$ then $0 < |x - 4| < \delta$

$$\Rightarrow |(7 - 3x) - (-5)| < \epsilon. \text{ Thus, } \lim_{x \rightarrow 4} (7 - 3x) = -5 \text{ by}$$

the definition of limit.

Ex-2.5

46) Use the intermediate value theorem to prove that there is a positive number c such that $c^2 = 2$. (This proves the existence of the number $\sqrt{2}$).

$\rightarrow f(x) = x^2$ is continuous on the interval $[1, 2]$, $f(1) = 1$, and $f(2) = 4$. Since $f(1) \neq f(2)$, there exist a number c in $(1, 2)$ such that $f(c) = c^2 = 2$; by the intermediate value theorem.

QUIZ #1
 CALCULUS 140
 Fall 2005, HUNSICKER

NAME _____ KEY _____ Honor Pledge _____

1) Define even and odd functions. Draw a picture or give an example to explain.

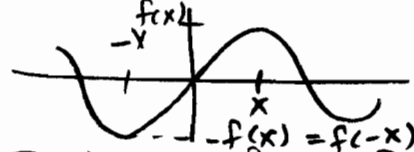
Def If $f(x)$ satisfies $f(x) = f(-x)$ for every number x in its domain then we say f is an even function. If $f(x) = -f(-x)$ for all x in its domain, we say f is an odd function.

idea Even functions are symmetric about the y -axis
 odd functions are symmetric about the origin.

Ex $f(x) = \cos x$ is even:



Ex $f(x) = \sin x$ is odd:



2) Let $f(x) = (1 + \frac{\ln 2}{x})^x$. Evaluate $f(x)$ for $x = 1, .5, .1, .01$ and $x = -1, -.5, -.1, -.01$. Use this information to guess $\lim_{x \rightarrow 0} (1 + \frac{\ln 2}{x})^x$.

x	$(1 + \frac{\ln 2}{x})^x$
1	1.69
.5	1.54
.1	1.23
.01	1.04
-.01	undefined
-.1	undefined
-.5	undefined
-1	3.25889

Since f is undefined for x close to 0 but negative, $\lim_{x \rightarrow 0} (1 + \frac{\ln 2}{x})^x$ doesn't exist.

However, from the values for $x > 0$, we can guess the right-handed limit is

$$\lim_{x \rightarrow 0^+} (1 + \frac{\ln 2}{x})^x = 1.$$