Midterm 1

Calculus 140
Fall 2005
Hunsicker

Notes: You have two hours to complete this exam. There are 100 points possible. Point totals for problems are indicated after problem numbers. You may use a calculator on this exam, but no other aids are permitted, such as notes, books, or classmates. Try to remember that your math exam is a form of communication. The more clearly you communicate to me, the better able I will be to evaluate and give proper credit for the thinking you did on each problem. Use full sentences when possible, and show the steps you made on the way to your solutions. If you have any questions, do not hesitate to come ask me about them during the exam. I may not answer your question, but it is better to ask than not.

I. THEORY

1. (5 points) Define rational function.
2. (5 points) Define continuity on an open and on a closed interval.
3. (5 points) State the Squeeze Theorem (you need not draw a picture or explain).
4. (10 points) State the Intermediate Value Theorem and explain what it says using a diagram and an informal definition.

5. (10 total) True/False: Write out "TRUE" or "FALSE" for each
   a. If f(x) is differentiable at x=a then it must be continuous at x=a.
   b. $\lim_{x\to\infty} f(x) = L$ means informally that as x gets bigger, f(x) gets closer to L, but never reaches it.
   c. The graph of a continuous function can always be drawn without lifting your pencil.
   d. If $\lim_{x\to a} f(x)$ exists, then the right and left handed limits must both exist.
   e. If $\lim_{x\to a} [f(x) + g(x)] = L$, then $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist and their sum is L.

II. WORD PROBLEMS

6. (15 points) The ideal gas law says that the pressure of 2 moles of gas at standard temperature (273.15 Kelvin) contained in an L liter container will be given by

$$P(V) = \frac{2 \cdot 8.314 \cdot 273.15}{V} \text{ kPa.}$$

a. For a certain industrial process, the pressure of this gas is to be maintained at 200 kPa (kilopascals). What volume container must it be contained in?
b. If there is an acceptable error of ∓ 5 kPa, what is the acceptable error in volume?
c. In terms of the precise definition of the limit, what value above corresponds to a? L? ε? δ?

7. (15 points) In the study of the seepage of irrigation water into soil, it may be found that the depth (in feet) to which water has seeped after t hours is given by f(t) = $\sqrt{t}$.

a. What happens to f(t) as t gets large? How would you write this mathematically?

b. What is the physical significance of f ′ (10)?

c. Recall that f ′ (t) = $1/2\sqrt{t}$. What happens to f ′ (t) as t gets large?

d. Try to explain why the answers in a and c make sense in this setting.

III. Calculations

8. (5 points) Write f(x) = $f(x) = \sqrt{x^3 + 3}$ as the composition of two functions.

9. (10 points) Find

$$\lim_{x \to 2} \frac{4x^2 - 16}{x - 2}.$$

10. (10 points) What must c be in the equation for f(x) below to make f(x) continuous at x=1?

$$f(x) = \begin{cases} 2x - 3 & x < 1 \\ x^2 + c & x \geq 1 \end{cases}$$

11. (10 points) Using the definition of derivative, find f ′ (t) if $f(t) = 2t^2 - 1$.