SOLUTIONS
MIDTERM #2, 2005
HUNSICKER

1) Def If $C(x)$ = cost to produce x units of a commodity then $C'(x)$ is called the marginal cost of production at production level x.

Since $C'(x) = \lim_{h\to 0} \frac{C(x+h)-C(x)}{h}$, by

letting $h=1$, we can see this is approximately equal to

$$C'(x) \approx C(x+1) - C(x)$$

This is the cost of producing one more unit if x are already being produced. For this reason, we also can think of marginal cost of production at production level x as the cost per additional unit produced if x are already being produced.

2) Def If f is a one to one function with domain A and range B, then the inverse of f is the function, denoted f^{-1}, with domain B and range A given by the rule

$$f^{-1}(y) = x \text{ if } f(x) = y.$$

3) a) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

b) $\lim_{x \to 0} \frac{\sin 5x}{3x} = \lim_{x \to 0} \frac{5}{3} \frac{\sin 5x}{5x}$

$= \frac{5}{3} \lim_{x \to 0} \frac{\sin 5x}{5x} = \frac{5}{3} \lim_{5x \to 0} \frac{\sin 5x}{5x}$

let $u = 5x$.

$= \frac{5}{3} \lim_{u \to 0} \frac{\sin u}{u} = \frac{5}{3} \cdot 1 = \frac{5}{3}$ by (a).

4) $f'(x) = 6x - 2 + \frac{1}{2\sqrt{x}}$ so the slope of the tangent line at $(1,2)$ is $f'(1) = 6 - 2 + \frac{1}{2} = 4\frac{1}{2} = \frac{9}{2}$.

Using point-slope form, then the equation of the tangent line is

$y - 2 = \frac{9}{2}(x-1)$

or we can write in slope-intercept as

$y = \frac{9}{2}x - \frac{5}{2}$

5) a) If height is $h(t) = -16t^2 + 8t + 8$
then velocity is
$v(t) = h'(t) = -32t + 8$
and acceleration is
$a(t) = v'(t) = h''(t) = -32.$

b) The max height occurs when $v(t) = 0$,
so $v(t) = -32t + 8 = 0$
$$32t = 8$$
$$t = \frac{1}{4}$$

c) The max height is its height at this time:
$$h\left(\tfrac{1}{4}\right) = -16\left(\tfrac{1}{4}\right)^2 + 8\cdot\left(\tfrac{1}{4}\right) + 8$$
$$= -1 + 2 + 8 = 9 \text{ feet high}$$

d) First we need to know <u>when</u> the ball hits the ground. This occurs when $h(t) = 0$:
$$0 = h(t) = -16t^2 + 8t + 8$$
$$0 = 2t^2 - t - 1$$
$$t = \frac{1 \pm \sqrt{1 - 4\cdot 2\cdot(-1)}}{2\cdot 2} = \frac{1 \pm \sqrt{9}}{4} = \frac{1 \pm 3}{4} = -\frac{1}{2} \text{ or } 1$$

We want $t > 0$ since $t < 0$ doesn't make sense in this problem. The speed at this time is $|v(1)| = |-32\cdot 1 + 8| = 24$ ft/sec.

6) $\frac{d}{dx}[x^2e^x] = \frac{d}{dx}[x^2]e^x + x^2\frac{d}{dx}[e^x]$

$$= 2xe^x + x^2e^x$$

7) $\frac{d^2}{dx^2}[\arctan(x)] = \frac{d}{dx}\left[\frac{d}{dx}[\arctan x]\right]$

$$= \frac{d}{dx}\left[\frac{1}{1+x^2}\right] = \frac{\frac{d}{dx}[1](1+x^2) - 1\frac{d}{dx}[1+x^2]}{(1+x^2)^2}$$

$$= \frac{0\cdot(1+x^2) - \left[\frac{d}{dx}[1] + \frac{d}{dx}[x^2]\right]}{(1+x^2)^2}$$

$$= \frac{-2x}{(1+x^2)^2}$$

8) $\frac{d}{dt}[\tan(\frac{1}{t})] = \frac{d}{dt}[\tan u]$ where $u = \frac{1}{t}$

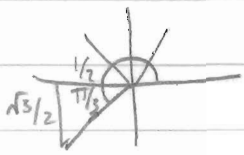
$$= \frac{d}{du}[\tan u]\cdot\frac{du}{dt} = \sec^2 u \cdot \frac{-1}{t^2}$$

$$= \frac{-\sec^2(\frac{1}{t})}{t^2}$$

at $t = (\frac{3}{4\pi})$,

$$= \frac{-\sec^2(1/(3/4\pi))}{(3/4\pi)^2} = \frac{-16\pi^2}{9}\cdot\sec^2(\frac{4\pi}{3})$$

$$= \frac{-16\pi^2}{9}\cdot\frac{1}{\cos^2(\frac{4\pi}{3})}$$



$\cos(\frac{4\pi}{3}) = -\frac{1}{2}$

$$= \frac{-16\pi^2}{9}\cdot\frac{1}{(-\frac{1}{2})^2} = \frac{-64\pi^2}{9}$$

9) $\frac{d}{dx}[3xy^2] = \frac{d}{dx}[x^2 - \cos y]$

$\frac{d}{dx}[3x]y^2 + 3x\frac{d}{dx}[y^2] = \frac{d}{dx}[x^2] - \frac{d}{dx}[\cos y]$

$3y^2 + 3x\frac{d}{dy}[y^2]\frac{dy}{dx} = 2x - \frac{d}{dy}[\cos y]\frac{dy}{dx}$

$3y^2 + 3x \cdot 2y\frac{dy}{dx} = 2x + \sin y\frac{dy}{dx}$

$6xy\frac{dy}{dx} - \sin y\frac{dy}{dx} = 2x - 3y^2$

$\frac{dy}{dx}[6xy - \sin y] = 2x - 3y^2$

$$\frac{dy}{dx} = \frac{2x - 3y^2}{6xy - \sin y}.$$

10) $\frac{d}{dx}\left[\ln\sqrt{\frac{e^x}{\sec x}}\right]$

$= \frac{d}{dx}\left[\frac{1}{2}\ln\frac{e^x}{\sec x}\right] = \frac{d}{dx}\left[\frac{1}{2}(\ln e^x - \ln\sec x)\right]$

$= \frac{d}{dx}\left[\frac{1}{2}(x + \ln(\cos x))\right]$

$= \frac{1}{2}\frac{d}{dx}[x + \ln(\cos x)] = \frac{1}{2}\left[\frac{d}{dx}[x] + \frac{d}{dx}[\ln(\cos x)]\right]$

$= \frac{1}{2}\left[1 + \frac{\frac{d}{dx}[\cos x]}{\cos x}\right] = \frac{1}{2}\left[1 + \frac{-\sin x}{\cos x}\right]$

$= \frac{1}{2} - \frac{1}{2}\tan x.$