PRACTICE FINAL ANSWERS

I. A) $\lim_{x \to a} f(x) = L$ if no matter how close you want $f(x)$ to be to L, you can get it that close (eg, within $\varepsilon > 0$) by choosing x sufficiently close to a (within δ, where δ is given in terms of ε).

B) If we want to get $3x-1$ within ε of 5, that means we want

$$5 - \varepsilon < 3x - 1 < 5 + \varepsilon.$$

Rearranging this, we get

$$6 - \varepsilon < 3x < 6 + \varepsilon$$
$$\frac{6}{3} - \frac{\varepsilon}{3} < x < \frac{6}{3} + \frac{\varepsilon}{3}$$
$$2 - \frac{\varepsilon}{3} < x < 2 + \frac{\varepsilon}{3}$$

So if x is within $\frac{\varepsilon}{3} = \delta$ of 2 then $3x-1$ is within ε of 5, thus $\lim_{x \to 2} 3x - 1 = 5$.

C) $e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ or $\lim_{t \to 0} (1+t)^{\frac{1}{t}}$

d) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x \to 2} x + 2 = 4$

we didn't do this stuff

e) $\lim_{x \to \infty} \frac{x^2}{e^x}$ is of the form $\frac{\infty}{\infty}$, so l'hopital's rule

$\Rightarrow \; = \lim_{x \to \infty} \frac{(x^2)'}{(e^x)'} = \lim_{x \to \infty} \frac{2x}{e^x}$ this is still $\frac{\infty}{\infty}$,

So apply l'hôpital again $= \lim_{x \to \infty} \frac{(2x)'}{(e^x)'} = \lim_{x \to \infty} \frac{2}{e^x} = 0$

II.

A) $$\left(\frac{x^2+2}{\sin x}\right)' = \frac{(x^2+2)'\sin x - (x^2+2)(\sin x)'}{\sin^2 x}$$

$$= \frac{2x\sin x - (x^2+2)\cos x}{\sin^2 x}$$

B) $$\left(\sqrt{\tan(3x)}\right)' = \left((\tan(3x))^{1/2}\right)'$$

$u = v^{1/2}$

$v = \tan w$ $\qquad \frac{du}{dx} = \frac{du}{dv}\frac{dv}{dw}\frac{dw}{dx}$

$w = 3x$

$$= \tfrac{1}{2}v^{-1/2} \cdot \sec^2 w \cdot 3$$

$$= \tfrac{1}{2}(\tan(3x))^{-1/2} \cdot \sec^2(3x) \cdot 3$$

$$= \frac{3}{2}\,\frac{\sec^2(3x)}{\sqrt{\tan(3x)}}$$

C) $$\left(e^{x^2+\ln x}\right)' = (x^2+\ln x)'\,e^{x^2+\ln x}$$

$$= \left(2x+\tfrac{1}{x}\right)e^{x^2+\ln x}$$

$$= \left(2x+\tfrac{1}{x}\right)e^{x^2}\cdot e^{\ln x}$$

$$= \left(2x+\tfrac{1}{x}\right)e^{x^2}\cdot x$$

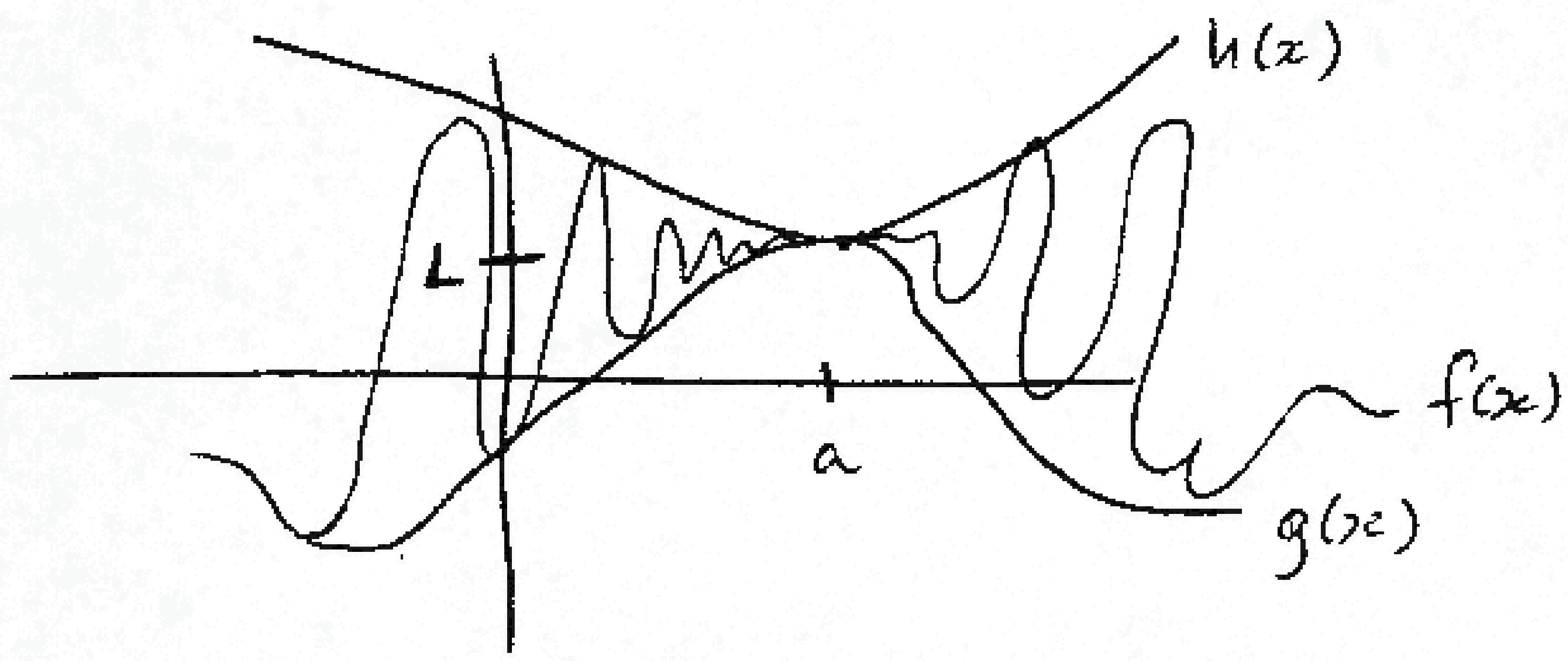
$$= (2x^2+1)e^{x^2}$$

III

we didn't do this

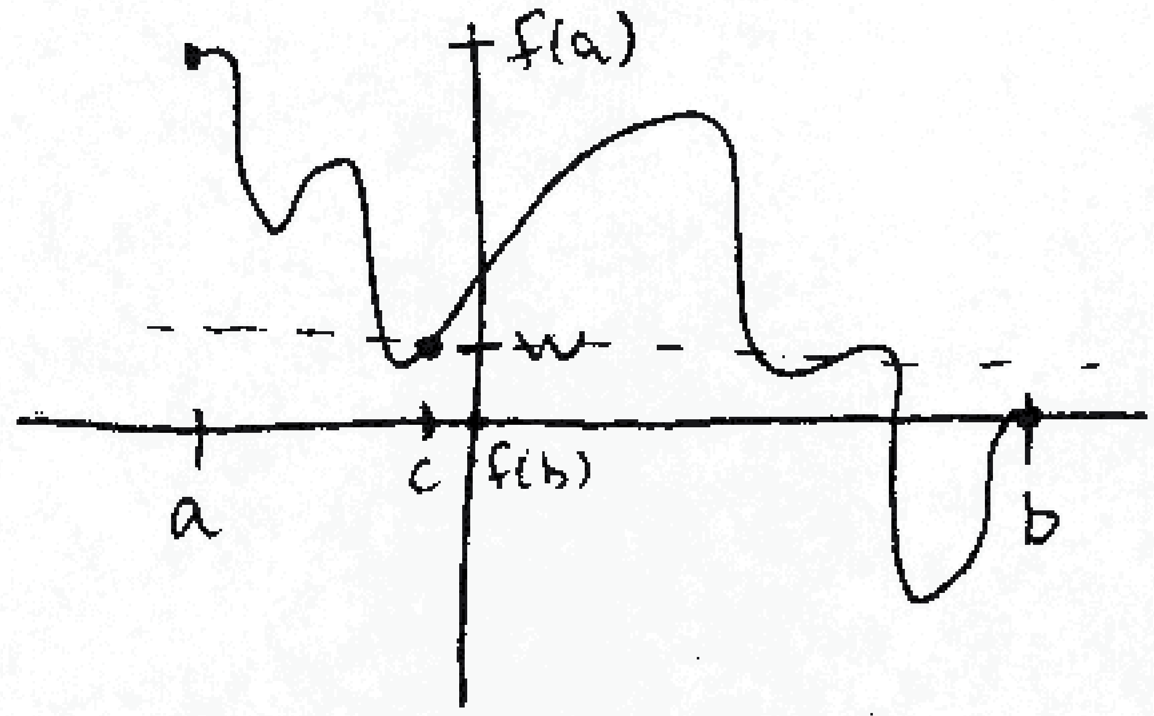
A) The squeeze theorem states that if $g(x) \le f(x) \le h(x)$ on an open interval containing a and if $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$,

then $\lim_{x \to a} f(x) = L$, as well



f(x) is "squeezed" near a to have limit L.

B) The Intermediate Value Theorem states that if f is continuous on a closed interval $[a,b]$ and if W is in between $f(a)$ and $f(b)$ then there is some $c \in [a,b]$ with $f(c) = W$.

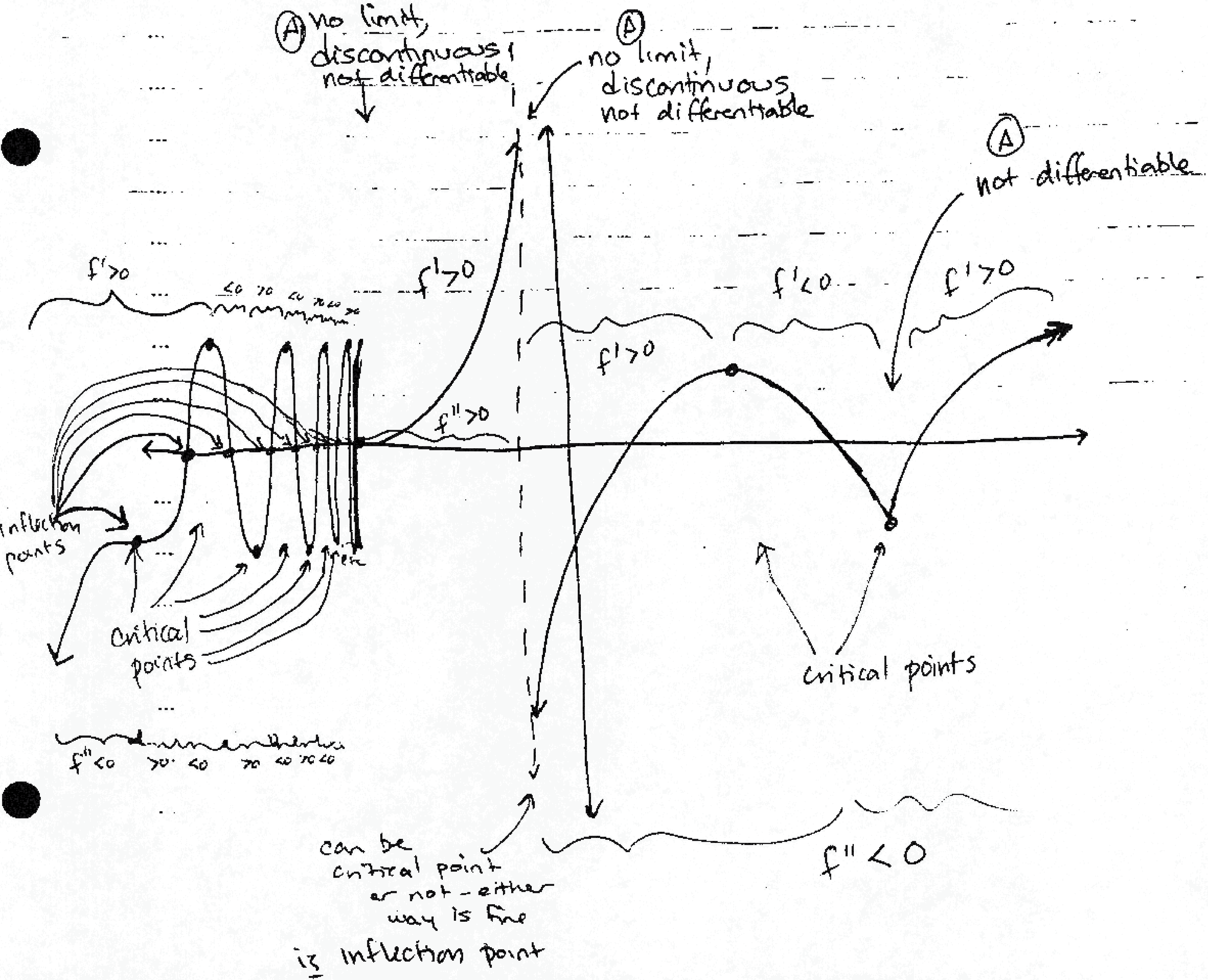


if f goes between f(a) and f(b) without pencil lifting, then it has to cross every intermediate value.

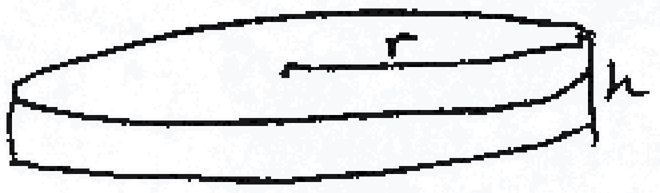
IV GRAPH (5 pts each)

A) INDICATE WHERE f is cont's, has limits, is differentiable.

B) Indicate where $f' > 0$, < 0, where $f''(x) > 0$, < 0. indicate critical points and inflection points



V

A) 

$1m^3 = \text{Volume} = \pi r^2 h$

$\frac{dh}{dt} = -.1 \text{ cm/hr} = -.001 \text{ m/hr}$

Question: What is $\frac{dr}{dt}$ when $r = 8m$?

implicitly differentiate $1 = \pi r^2 h$ with respect to t:

$$0 = \pi\left(2r\frac{dr}{dt}h + \pi r^2 \frac{dh}{dt}\right)$$

Plug in values when $r = 8m$: $\pi \cdot 8^2 h = 1 \Rightarrow h = \frac{1}{8^2\pi}$

$$0 = \pi\left(2\cdot\cancel{8}\cdot\frac{dr}{dt}\cdot\frac{1}{8^2\pi} + \pi(8)^2(-.001)\right)$$

$$0 = \frac{1}{4}\frac{dr}{dt} - .064\pi$$

$$.064\pi = \frac{1}{4}\frac{dr}{dt}$$

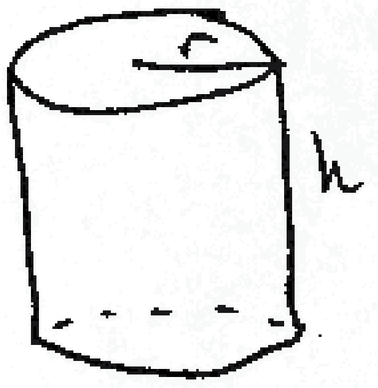
$$4 \cdot .064\pi = \frac{dr}{dt}$$

$$\begin{array}{r} {}^{1} \\ 64 \\ 4 \\ \hline 256 \end{array}$$

$$\frac{dr}{dt} = .256 \text{ m/hr}$$

V

B)



$\pi r^2 h = \text{Volume} = 100\,cm^3$

$2\pi r^2 =$ area of top + area bottom

$2\pi r h =$ area of sides

$2c =$ cost of top & bottom /unit

$c =$ cost of sides /unit (constant)

$2\pi r^2 \cdot 2c + 2\pi r h \cdot c =$ total cost of can

Make cost a function of r only:

$h = \frac{100}{\pi r^2}$, so

$$\text{cost}(r) = 2\pi r^2 \cdot 2c + 2\cancel{\pi}\cancel{r} \cdot \frac{100}{\cancel{\pi} r^{\cancel{2}}} \cdot c$$

$$\text{cost}(r) = 4\pi r^2 c + \frac{200c}{r}$$

Critical points will be endpoints and places where $\text{cost}'(r) = 0$ or does not exist.

The domain for r is $0 < r < \infty$, so there are no endpoints.

$\text{cost}'(r) = 8\pi r c - \frac{100c}{r^2}$, which is defined on $0 < r <$

so we just need points where $\text{cost}'(r) = 0$:

$$0 = 8 r c \pi - \frac{100c}{r^2} \Rightarrow 8\pi r c = \frac{100c}{r^2} \Rightarrow$$

$$r^2 \cdot 8\pi r c = 100c \Rightarrow r^3 = \frac{\cancel{100}^{25}\,\cancel{c}}{\cancel{8}_{2}\,\pi\,\cancel{c}} = \frac{25}{2\pi}$$

$r = \sqrt[3]{\frac{25}{2\pi}}$, $h = \frac{100}{\pi\left(\frac{25}{2\pi}\right)^{2/3}}$ are the dimensions

of the least expensive can.