CALCULUS 140
MIDTERM #1
SELECTED SOLUTIONS

1) LIMITS

1) Make sure to include all 5 parts of this essay that we discussed in class

- Formal definition (a complete one)
- Diagram or graph with all symbols that occur in definition
- Intuitive idea of definition
- Explanation of how the intuitive idea is expressed in the formal definition
- Examples where the definition does not apply with explanations of why

2) Similar to above, but include

- Complete statement of theorem
- diagram or graph with all symbols which appear in the theorem
- Intuitive idea
- List of hypotheses & conclusions
- Examples where the hypotheses fail and so, therefore, do the conclusions

3) a) $\lim_{x \to -1} x^5 + 2x^3 - \sqrt{(-4x)}$

$= \lim_{x \to -1} x^5 + \lim_{x \to -1} 2x^3 - \lim_{x \to -1} \sqrt{4x}$ by sum & difference rule

$= (-1)^5 + 2 \lim_{x \to -1} x^3 - \sqrt{\lim_{x \to -1} (-4x)}$ by power, const mult, power rule since $\lim_{x \to -1} -4x = 4 > 0$

$$= -1 + 2(-1)^3 - \sqrt{4} = -5$$

power rule, const multiple rule

b) $$\lim_{x\to 5} \frac{x^2-x-20}{x-5} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+4)}{\cancel{x-5}} = \lim_{x\to 5} x+4 = 9$$

by substitution

since limits don't care about $x=5$

c) $$\lim_{x\to\infty} \frac{5x^3-100x^2+2}{2x^3+17}$$

$$= \lim_{x\to\infty} \frac{5-\frac{100}{x}+\frac{2}{x^3}}{2+\frac{17}{x^3}}$$ (multiply by $\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$)

$$= \frac{\lim_{x\to\infty}\left(5-\frac{100}{x}+\frac{2}{x^3}\right)}{\lim_{x\to\infty}\left(2+\frac{17}{x^3}\right)}$$

by quotient rule for limits, now that neither top nor bottom goes to ∞

$$= \frac{5}{2}$$

substitution & addition & sub. rules

d) $$\lim_{x\to 1} \frac{|x-1|}{(x-1)}$$ does not exist, as

$$\lim_{x\to 1^+} \frac{|x-1|}{x-1} = \lim_{x\to 1^+} \frac{x-1}{x-1} = \lim_{x\to 1^+} 1 = 1$$

whereas

$$\lim_{x\to 1^-} \frac{|x-1|}{x-1} = \lim_{x\to 1^-} \frac{-(x-1)}{x-1} = \lim_{x\to 1^-} -1 = -1$$

II Continuity

4) Use class definition - essay not required

5) $f(x) = x^2 - 2$ is a polynomial, so by substitution, $\lim_{x \to a} x^2 - 2 = f(a) = a^2 - 2$ for all

a in the domain of f, which is all of $\mathbb{R}$. So for all $a \in \mathbb{R}$, $\lim_{x \to a} x^2 - 2 = a^2 - 2$, ie, $x^2 - 2$ is continuous ~~on~~ the entire interval $\mathbb{R}$. Thus it is continuous on any closed subinterval, eg $[0,3]$.

6) The IVT says if $f(x)$ is continuous on $[a,b]$ and N is between $f(a)$ and $f(b)$, then there is some $c \in (a,b)$ with $f(c) = N$. So here, let $f(x) = x^2 - 2$, $a = 0$, $b = 3$ and $N = 0$, which is between $f(a) = f(0) = -2$ and $f(b) = f(3) = 7$. We can apply IVT by 5) to find $c \in (0,3)$ with $f(c) = N = 0$ ie, $c^2 - 2 = 0$, so $c^2 = 2$. This c is the required $\sqrt{2}$.

7) $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 2x + 3 = 2(-1) + 3$ by substitution
$= 1$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} -x + 2 = -(-1) + 2 = 3$ by substitution

$1 \neq 3$ so the 1-sided limits disagree, and there is no limit at -1, so f is not continuous there.

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} -x+2 = -1+2 = 1$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 - 2x + 2 = 1^2 - 2 \cdot 1 + 2 = 1$$

The two sided limits agree, so $\lim_{x \to 1} f(x) = 1$.
Furthermore, $f(1) = -1+2 = 1 = \lim_{x \to 1} f(x)$
so f <u>is</u> continuous at 1.

III DERIVATIVES

8) Same comments as 1)

9) By definition, if $f(x) = x^2+3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^2+3] - [x^2+3]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+3} \cancel{-x^2} \cancel{-3}}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \lim_{h \to 0} 2x+h = 2x$$

So $\boxed{f'(x) = 2x}$

10) This was somewhat misleading, so I accepted 2 answers

1) $c'(10)$ is the amount being consumed per year in 2000 (if $c(t)$ = amount consumed (in millions of barrels) from 1990 to $1990+t$).

2) $C'(10)$ is the rate at which yearly oil consumption is increasing in 2000 (if $C(t)$ = amount consumed in the t^{th} year after 1990 (in millions of barrels/yr).

To estimate $C'(10)$ make a table

h	$10+h$	$C(10+h)$	$C(10+h)-C(10)$	$\frac{C(10+h)-C(10)}{h}$
1	11	202.312	26.243	26.243
.1	10.1	178.631	2.56282	25.6282
.01	10.01	176.324	.255653	25.5653
		$C(10) = 176.068$		
-.01	9.99	175.813	-.255513	25.5513
-.1	9.9	173.519	-2.5488	25.488
-1	9	151.228	-24.8399	24.8399

So $C'(10) \approx 25.5$

IV GRAPH READING:

a) Discontinuous either where limit does not exist or ≠ value, so
-2, -1, 0, 2, 3

b) Not differentiable where not continuous or where there is a corner, so -3, -2, -1, 0, 2, 3

c) $\lim_{x \to -2} f(x) = 2$ $\lim_{x \to \infty} f(x) = 2$

$\lim_{x \to 2^-} f(x) = 3$ $f(3) = 1$