

MIDTERM #1
CALCULUS 140, FALL 2003

NAME _____ HONOR PLEDGE _____

NOTES: This is a two hour exam. Point totals are written beside the problems. Allot your time accordingly. There are 111 points possible on this exam. It will be graded out of 100. Calculators are allowed on this exam, but no books or notes may be used.

I. LIMITS

- 1) (10 points) Use the definition of $\lim_{x \rightarrow a} f(x) = L$ (that is, the one involving ϵ and δ) to prove that $\lim_{x \rightarrow 3} 2x + 1 = 7$.
- 2) (10 points) State the Squeeze Theorem. Draw a graph that illustrates the idea behind this theorem, and explain it briefly.
- 3) (5 points each) Find each indicated limit by any means if it exists. Explain your work. If any limit does not exist, explain why:

a) $\lim_{x \rightarrow -1} x^5 + 2x^3 - \sqrt{-4x}$

b) $\lim_{x \rightarrow 5} \frac{(x^2 - x - 20)}{(x - 5)}$

c) $\lim_{x \rightarrow \infty} \frac{(5x^3 - 100x^2 + 2)}{(2x^3 + 17)}$

d) $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1}$

II. CONTINUITY

- 4) (10 points) Define what it means for a function to be continuous on a closed interval.
- 5) (5 points) Show using the definition in 4) and any limit theorems you know that the function $f(x) = x^2 - 2$ is continuous on the interval $[0,3]$.
- 6) (5 points) You are probably accustomed to writing $\sqrt{2}$ as if you knew for certain that there is a positive number whose square is 2. Prove that such a number does exist using the Intermediate Value Theorem and problem 4.
- 7) (5 points) Let
$$f(x) = \begin{cases} 2x + 3 & x < -1 \\ -x + 2 & -1 \leq x \leq 1 \\ x^2 - 2x + 2 & x > 1 \end{cases}$$
Determine if $f(x)$ is continuous at the points $x = -1$, $x = 1$.

III. DERIVATIVES

- 8) (10 points) Define the derivative of $f(x)$. Draw a diagram to illustrate its meaning, and explain in a paragraph why the definition has the intuitive meaning it has in terms of the graph of $f(x)$.
- 9) (10 points) Use the definition in 8) to find $f'(x)$ for $f(x) = x^2 + 3$.
- 10) (10 points) Suppose the U.S. consumption of oil (in millions of barrels) t years after 1990 is given by the function $C(t) = \sqrt{30t^3 + 1000}$. How would we interpret $C'(10)$? Make a table of difference quotients for $h = -1, -0.1, -0.01, 0.01, 0.1, 1$ and use this to estimate $C'(10)$.

IV. GRAPH READING

Given the graph below, find each of the following if they exist or state that they do not:

- a) (4 points) All points where $f(x)$ is NOT continuous
- b) (4 points) All points where $f(x)$ is NOT differentiable
- c) (2 points each) $\lim_{x \rightarrow -2} f(x)$
 $\lim_{x \rightarrow \infty} f(x)$
 $\lim_{x \rightarrow 2^-} f(x)$
 $f(3)$

