NO AIDS ALLOWED ON EXAM

CALCULUS 14A FINAL EXAM

NAME _____

ID # _____

I. DERIVATIVES (5 pts each / 30 total)

A) DEFINE DERIVATIVE

B) USE THE DEFINITION TO FIND THE DERIVATIVE OF $\frac{1}{x}$.

C) FIND THE FOLLOWING DERIVATIVES:

i) $(x^2 \tan x)'$

ii) $(\sin^2(x^2+3))'$

iii) $\left(\ln\left(\frac{3x^4\sqrt[3]{x^2+1}}{\sqrt{x^3+2}}\right)\right)'$

D) DEFINE ANTIDERIVATIVE

II. LIMITS (5 pts each / 10 total): EVALUATE:

A) $\lim_{x \to 0} \frac{\sin 3x}{2x}$

B) $\lim_{x \to 0} \frac{x^3}{x - \sin x}$

III. STATE THE MEAN VALUE THEOREM AND DRAW AN EXPLANATORY PICTURE (5 pts)

IV. GRAPH: (5 PTS EACH/10 TOTAL)

INDICATE ON THE GRAPH BELOW

A) WHERE f HAS NO LIMIT

" " IS NOT CONTINUOUS

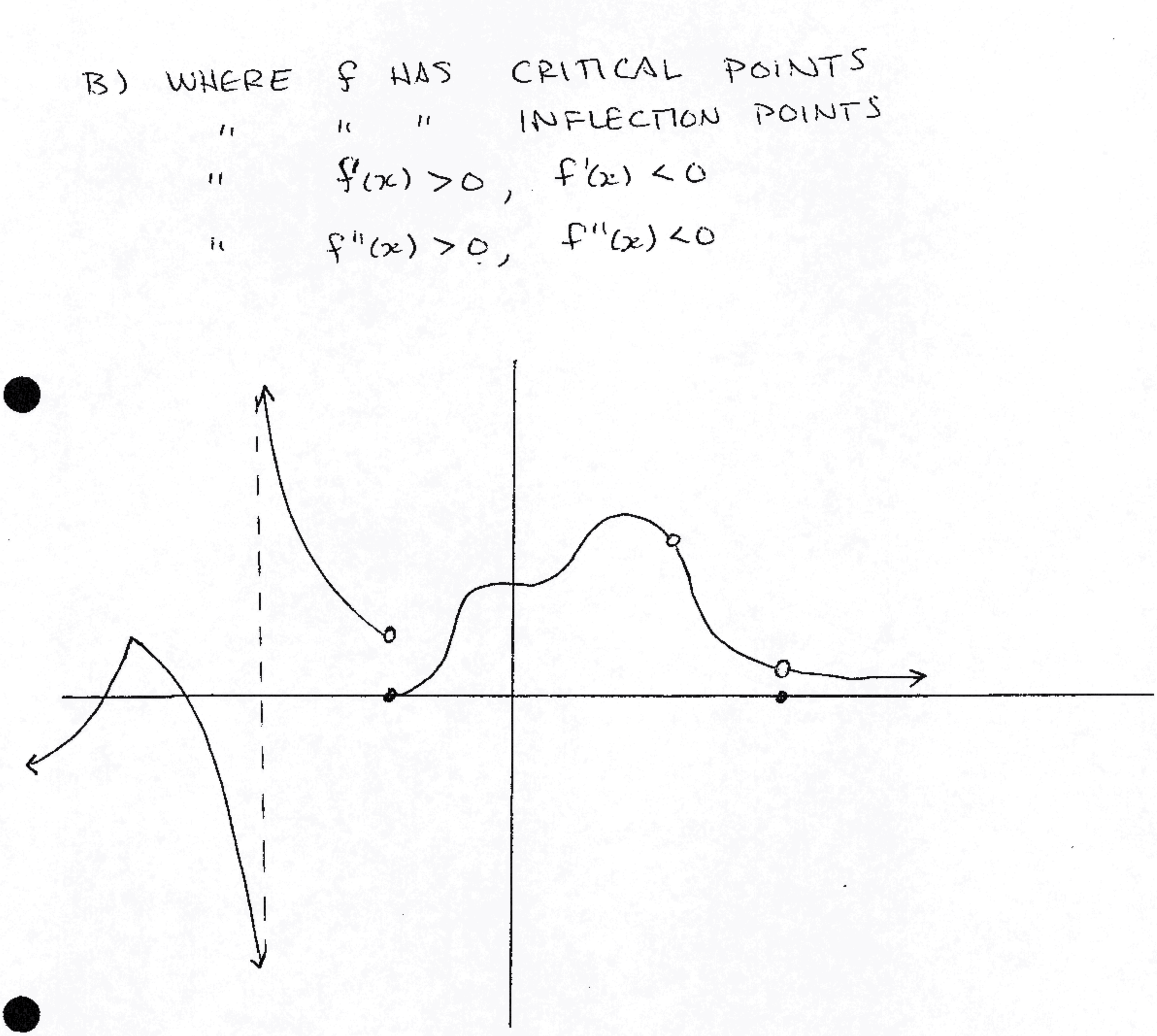
" " HAS NO DERIVATIVE

B) WHERE f HAS CRITICAL POINTS

" " " INFLECTION POINTS

" $f'(x) > 0$, $f'(x) < 0$

" $f''(x) > 0$, $f''(x) < 0$



V WORD PROBLEMS (10 pts each / 20 total)

A) A model rocket is launched from the roof of my elementary school, which is 30 ft high, at a velocity of 64 ft/sec. How long does it take to reach its highest point? How high does it go? (my brother used to do this when I was a kid)

B) Two noisy fraternities are located 1000 feet away from each other. If one is [redacted] times as noisy as the other, and if the intensity of noise at a distance of x feet from the source is given by

$$\text{Intensity} = \frac{\text{noisiness}}{x^2},$$

what is the quietest place along the street between them? You may assume the less noisy frat has a noisiness level of 1.

VI INVERSE FUNCTIONS (5 pts)

USE THE INVERSE FUNCTION DERIVATIVE THEOREM TO FIND THE DERIVATIVE OF ARCSIN(x).

VII THE INTEGRAL, FUNDAMENTAL THEOREM (5 pts each)

A) APPROXIMATE THE AREA UNDER THE GRAPH OF $f(x) = x^2 + x$ BETWEEN $x = 2$ AND $x = 4$ BY DIVIDING THE INTERVAL INTO 4 PIECES AND TAKING UPPER AND LOWER ESTIMATES

B) SET UP THE SUMS (YOU NEED NOT SOLVE) WHICH DESCRIBE THE UPPER & LOWER ESTIMATES IF YOU DIVIDE THE INTERVAL INTO 12 PIECES

C) STATE THE FUNDAMENTAL THEOREM OF CALCULUS

D) (5 pts EXTRA CREDIT)
FIND THE PRECISE AREA OF THE REGION IN A).

I'll get solns on reserve tomorrow (Tues)