

Calculus 140, Final Exam

Fall 2002, Hunsicker

Each numbered problem is worth 5 points unless otherwise indicated. There are a total of 100 points possible, plus 5 points of extra credit.

0.1 Limits

1) (6 points) Draw a function with all of the following properties:

$$\begin{aligned} \lim_{x \rightarrow \infty} f(x) = 2, & \quad \lim_{x \rightarrow 0^+} f(x) = 5, & \quad \lim_{x \rightarrow -2} f(x) = \infty, \\ \lim_{x \rightarrow -\infty} f(x) = -\infty, & \quad \lim_{x \rightarrow 0^-} f(x) = -1, & \quad \lim_{x \rightarrow 3} f(x) = 1 \end{aligned}$$

2) (4 points) Determine the following limit or explain why it does not exist:

$$\lim_{x \rightarrow 2} \frac{(x-2)^2}{|x-2|}$$

0.2 Continuity

3) Define continuity on a closed interval.

4) State the Intermediate Value Theorem and explain it with a picture.

0.3 The Derivative

5) State the definition of the derivative.

6) Explain its meaning in terms of population by examining the various components of the definition.

7) Use the definition to find $\frac{d}{dx}(\sqrt{x})$.

8) Show that if $f(x)$ and $g(x)$ have the same derivative, then they differ by a constant.

0.4 Applications of the Derivative

9) (10 points) A company has the option of scheduling a laborer for up to 4 hours per day of overtime work (at \$15/hour) in addition to the standard 8 hour workday at (\$10/hour). A certain job requires 200 person-hours to complete, and only one laborer is available for the job. If the company incurs \$200 of costs for

each day that the job remains unfinished, how many hours per day should the laborer work to minimize the cost?

10) (10 points) The velocity at which a viscous fluid flows through a circular tube is not the same at all points of a cross-section, and decreases to zero at the walls. For a point at radial distance r from the center of the tube, the velocity of the flow is

$$v = \frac{\alpha}{L}(R^2 - r^2),$$

where L is the length of the tube, R is the total radius of the tube, and α is a constant related to properties of the fluid and tube walls. Assume that $L = 25$ and $\alpha = 1$. What is the acceleration of the fluid moving at the center of the tube when the total radius is 10 cm if the total radius is increasing at a rate of .02 cm/minute?

0.5 Exponents and Logarithms, Inverse Functions

11) Prove that $\log_b xy = \log_b x + \log_b y$ using the fact that $b^a b^c = b^{a+c}$.

12) When a foreign substance is introduced into the body, the body's defense mechanisms react by breaking down the substance and excreting it. The rate of excretion is proportional to the concentration of the substance in the body. If, after 12 hours, 30% of a certain substance has been excreted, how long will it take until half of it has been excreted?

13) Prove that $\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$.

0.6 The Riemann Integral and the Fundamental Theorem

14) Define Riemann Integral

15) Assume that a certain 1 meter long rod has density given by t^2 kg/meter, measuring from one end. Set up a Riemann sum with five partition points $t_0 < t_1 < t_2 < t_3 < t_4$ which estimates the mass of this rod. Explain why this sum approximates the mass of the rod.

16) Find its exact mass by finding $\int_0^1 t^2 dt$ using Riemann Sums.

17) State the Fundamental Theorem of Calculus, first version.

18) Find the exact mass of the rod in 15 by using the second version of the Fundamental Theorem of Calculus.

0.7 Extra Credit

Give an example of a quantity that undergoes exponential growth or decay. Explain the scientific principle which governs the growth or decay of this quantity, and why that law can be expressed mathematically as the differential equation for growth and decay.