Calculus 140, Fall 2002
Prof. Hunsicker

MIDTERM EXAM #2

1) (5+5+10+10) Antiderivatives and motion
   a) Define antiderivative.
   b) If $f(t)$ and $g(t)$ are both antiderivatives for $h(t)$ on a given interval, what can say about them? Draw a picture to explain.
   c) The acceleration due to gravity on Mars is 3.7 m/s^2. Explain how your answers to parts a) and b) allow you to determine the height of an object falling on Mars as a function of time. What besides acceleration do you need to know?
   d) If the first astronaut on Mars drops a rock hammer from a height of 1.5 meters, how much longer will it take to hit the ground than it would have back on Earth (where gravity is 9.8 m/s^2)?

2) (10 points) Pick *one* of the following quantities and explain, using the difference quotient definition of the derivative (i.e., the usual one), why this quantity can be expressed as (or approximated by) a derivative: density, magnification, marginal cost.

3) (10 points) The population of carp in the Fox River depends on the PCB concentration in the river. Let y represent the number of carp per mile of river, and x represent PCB concentration in parts per million (ppm). If the PCB concentration is increasing at a rate of 20 ppm/year and population of carp as a function of PCB pollution is given by the equation:

$$y=1000/(1+x)$$

how fast is the carp population decreasing when the PCB level is at 200ppm?

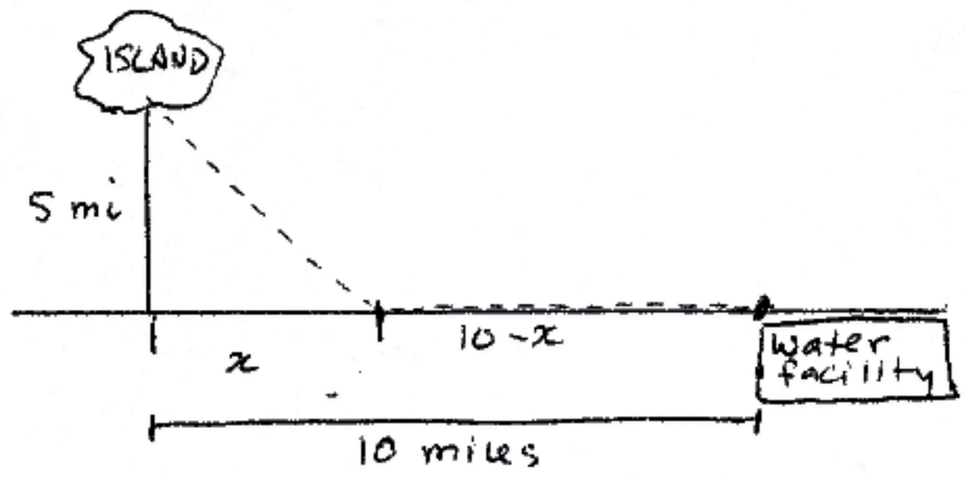
4) (5 points each) Set up BUT DO NOT SOLVE the following problem according to the steps below:

Dudley's Delicious Apples determines that the annual yield per apple tree is fairly constant at 352 pounds when the number of trees per acre is 55 or fewer. For each additional tree over 55, the annual yield per tree decreases by 5 pounds, due to overcrowding. How many trees should be planted per acre to maximize the annual yield per acre?

   a) What quantity are we trying to optimize in this problem? What other variables are relevant? Clearly define each one, and circle the one we are trying to optimize.
   b) How are these variables related (one or more equations)?
   c) Write the quantity to be optimized in terms of just one variable.

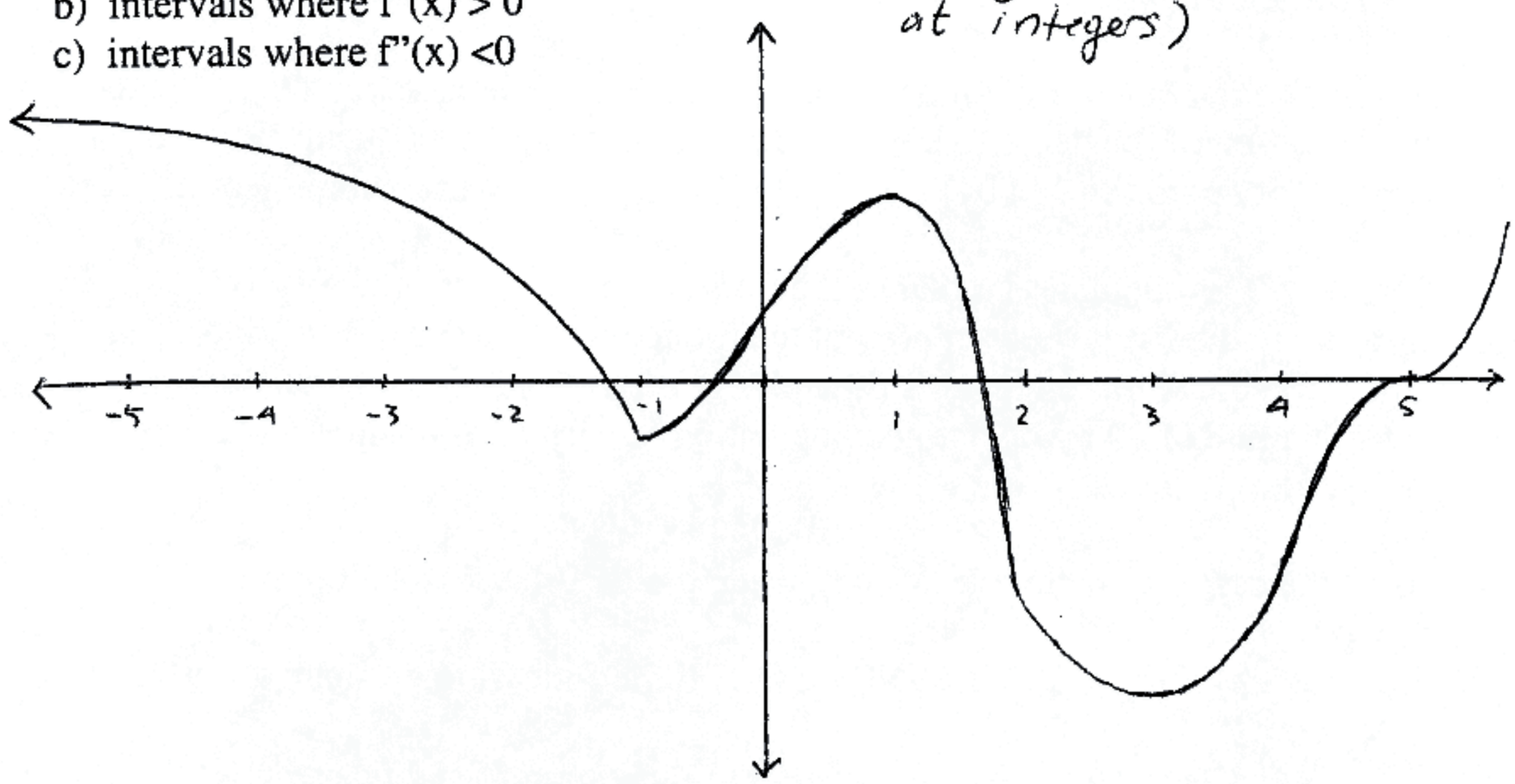
5) (5+10+10)
   a) State the Theorem of the Interior Extremum.
   b) What are the critical values of a function on a closed interval, and how do we use them for optimization?
   c) Use this to solve the following problem: A freshwater pipeline is to be run from a

source on the edge of a lake to a small resort community on an island 5 miles offshore, as indicated in the figure below. The cost of the pipeline for a given x is $C(x)=11{,}000\cdot(25+x^2)^{1/2} + 10{,}000(10-x)$. What choice of x minimizes the cost of the pipeline?



6) (10 points) Referring to the graph of f(x) below, identify all
   a) points where f'(x)=0
   b) intervals where f'(x) > 0
   c) intervals where f''(x) <0

(assume that any relevant stuff happens at integers)



Extra Credit (5 points)

When we considered the ladder being pulled away from the wall at a constant rate, we found that as we had set the model up, the top of the ladder hit the ground infinitely fast, which is not possible. How and where does this model break down? Assume that the ladder is attached to the wall somehow at the top in a manner that allows it to slide freely up and down.