

Calculus 140, Midterm #1
October 21, 2002
Prof. Hunsicker

This exam is worth 100 points. Point values are indicated for each problem. Make sure you show all work, including explaining if and how you have used your calculator. Remember to write your name on all blue books you use, and sign the honor code on the first one. No books or notes are permitted on this exam.

1) (5 points each)

- State the precise definition of $\lim_{x \rightarrow a} f(x) = L$.
- Prove that $\lim_{x \rightarrow 2} 2x + 1 = 5$ using the definition. (Hint: let $\delta = \epsilon/2$.)
- How would you change the definition in part a) to define $\lim_{x \rightarrow a^+} f(x) = \infty$?

2) (10+5 points)

- State the Intermediate Value Theorem and draw a picture to explain what it means.
- Use it to show that the function $f(x) = 2^x - 5$ has a root in the interval $[0,3]$.

3) (5+10+10 points)

- State the formal definition of the derivative.
- What geometric meaning does the derivative of $f(x)$ at a point have? Why? Use a picture to explain.
- Use the definition of the derivative to find $f'(4)$ when $f(x) = x^2 + 1$.

4) (5 points each) Find each of the following limits or explain why it does not exist:

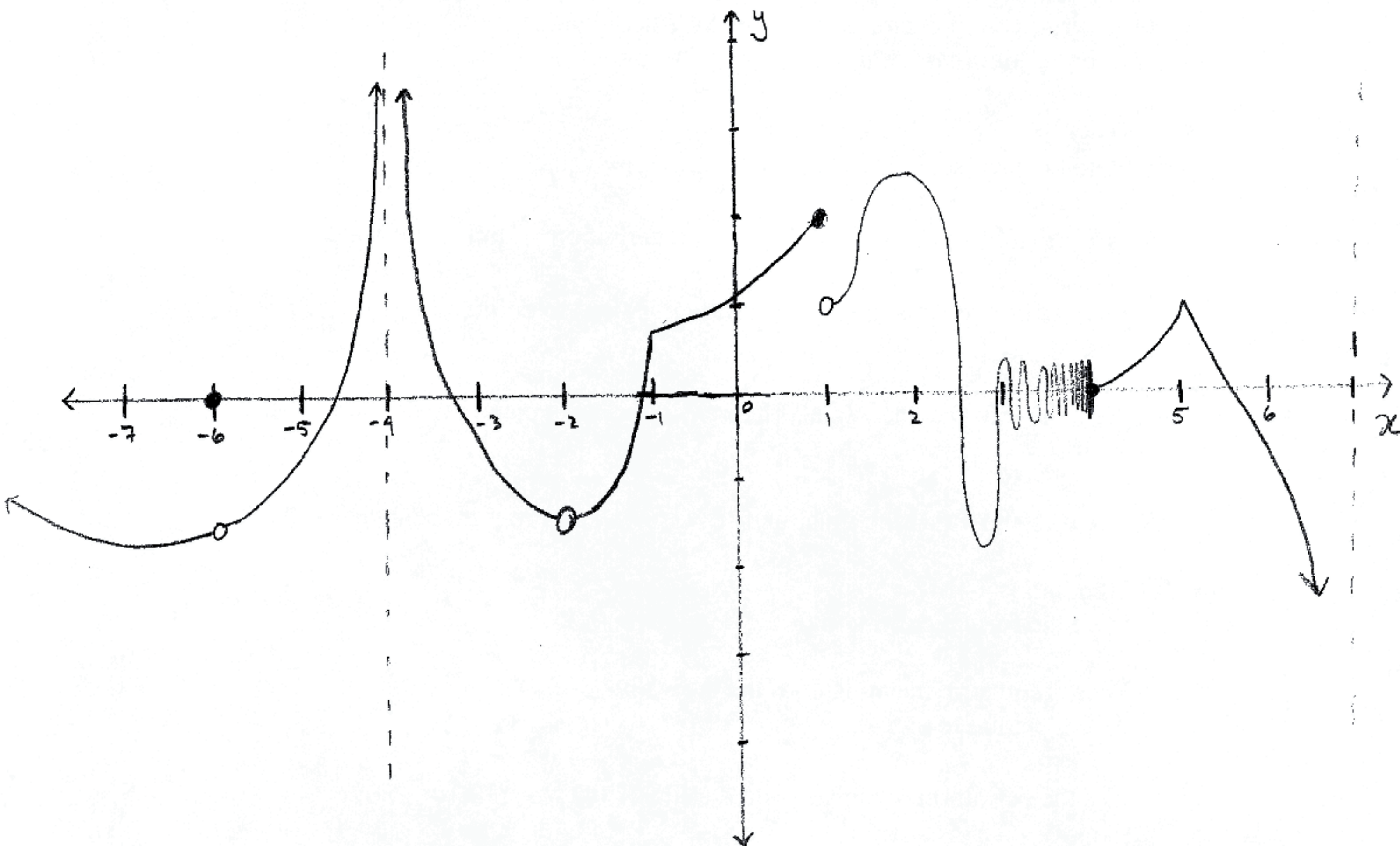
a) $\lim_{x \rightarrow 2^+} \frac{x^2 - 4}{x - 2}$ b) $\lim_{x \rightarrow 0} \frac{|x|}{x}$ c) $\lim_{x \rightarrow \infty} \frac{x^2 + 3x + 1}{2x^2 + 4}$

5) (5 points each) Using the sum, difference, product, quotient, power, sin and cos rules, calculate each of the following derivatives:

a) $(\cot(x))'$ b) $f'(\pi)$ when $f(x) = x \sin(x)$ c) $f'(x)$ when $f(x) = \frac{x}{5x^3 - 2x + 3}$

6) (10+5 points)

- a) The function $f(x)$ is graphed below. State where $f(x)$ does not have a limit, where $f(x)$ is not continuous, and where $f(x)$ is not differentiable. Give a brief explanation of why in each case.



- b) The function above is not continuous on the interval $[1, 4]$, but it does attain both a maximum and a minimum on this interval. Why doesn't that contradict the Maximum and Minimum Value Theorems?