Disagreement and Evidence Production in Pure Communication Games*

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Abstract

We expand the Crawford-Sobel (1982) model of information transmission to allow for the costly provision of "hard evidence" in addition to free "soft signals" (i.e., conventional cheap talk). We prove the existence of an interval-partition equilibrium, where each cheap-talk message is sent by an interval of Sender-types, while hard signals are sent by types belonging to a finite union of intervals. We also show that the availability of costly hard signals may reverse one of the important implications of the classical cheap talk model, namely, that diverging preferences always lead to less communication.

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1 Introduction

In his work on persuasion, Aristotle (347 B.C.E.) argues that a trained rhetor can rely on three fundamental tools in order to convince his audience: *Logos* (evidence and deduction), *Ethos* (speaker credibility) and *Pathos* (listener emotions). This classification seems appropriate in many economic interactions involving communication. For example, an expert advising a decision maker on policy can use hard data, or make unverifiable statements, or simply appeal to the decision maker's emotions. A fundamental question of rhetoric is: What mixture of these three tools of persuasion is most effective in different situations?

In game theoretical models of communication (sender-receiver games) *Logos* corresponds to verifiable, hard information, *Ethos* to unverifiable pure cheap talk, and *Pathos* remains to be quantified.[1] In this paper, we investigate certain tradeoffs between hard and soft information transmission (*Logos* and *Ethos*) in a basic pure communication game à la Crawford and Sobel (1982). Our most interesting results concern the change in the use and optimal mixture of hard and soft signals, and the amount of information transmitted in equilibrium, as the interests of the expert and the decision maker become less aligned.

We consider a model of information transmission where the costly provision of "hard evidence" is available in addition to free "soft signals" (i.e., conventional cheap talk). We prove the existence of an interval-partition equilibrium, where each cheap-talk message is sent by an interval of the sender's types, while hard signals are sent by types belonging to a finite union of intervals. We also show that the availability of costly hard signals can reverse one of the most robust implications of the classical cheap talk model, namely, that diverging preferences always lead to less communication.

[1] A hard signal sent by the sender is evidence that a particular state of nature has obtained, and so can be used by the receiver as a basis for logical inferences (*Logos*). The meaning of a cheap talk message in equilibrium depends on the set of sender types who send that message, and therefore can be interpreted as deriving from the reputation or credibility of the sender (*Ethos*). We are unaware of communication models where the players' emotions (*Pathos*) are explicitly taken into account.

Our model and results are relevant in a number of applications. By characterizing equilibria where both cheap talk messages and costly hard signals are used, we show how formal evaluations and informal assessments can coexist in education, management control, and firms' internal communication systems. We argue that when evidence production is costly, it is the most extreme types of the sender who provide hard signals in order to avoid being pooled with moderate types. Most interestingly, in certain cases an *increase in the conflict* between the expert's and the decision maker's preferences leads to a heavier reliance on hard evidence. As a result, *more information* is transmitted in equilibrium and the decision maker is better off.

An area where cheap talk games have been applied particularly successfully is the study of parliamentary institutions. In many political systems legislation is proposed by committees that are better informed about policy consequences than the median voter in the legislature. The leading theoretical framework in which this legislative process is formulated, Gilligan and Krehbiel's (1987, 1989) model, is formally equivalent to Crawford and Sobel's (1982) sender-receiver game with a uniform type distribution, constant bias and quadratic utilities. The standard model only allows for the transmission of "soft" information in the form of a proposal from the committee to the floor. But in reality committees can also produce reports, transcripts of hearings and expert testimony in addition to a proposed bill. The provision of this type of "hard evidence" is clearly costly for the committee; for example they need resources to summon and interview witnesses. This is exactly the feature that our model attempts to capture. Contrary to received wisdom, our results indicate that a committee whose median member is more "biased" relative to the median legislator may provide more information to the legislature in the form of hard evidence, which leads to better decisions from the perspective of the median legislator.

Our model considers free and unverifiable as well as costly and verifiable signals in a communication setting. The canonical model of the transmission of unverifiable information is that of Crawford and Sobel (1982) with its well-known prediction regarding partition (imperfectly separating) equilibria.

In a similar setup, Grossman (1981) and Milgrom (1981) show that when information is verifiable, the unique equilibrium involves perfect separation of all sender types.[2] Their unraveling argument applies when the sender can send hard signals at no cost. In a similar vein, Bull and Watson (2004, 2007) argue that courts and mechanisms should only accept positive (direct) proofs of claims made by informed parties when such disclosures are available.

Verrecchia (1983) points out that if disclosure of hard information is *costly* then the full-revelation results of Grossman (1981) and Milgrom (1981) may not apply. In Verrecchia's model a firm's management can either verifiably disclose (at a certain cost) the firm's profitability, or refuse to do so. The market has rational expectations and values the firm's shares according to the disclosure (or lack thereof). In equilibrium the firm discloses its profitability if and only if it is above a certain positive threshold and withholds the information otherwise. At the threshold the firm is indifferent between being valued fairly while incurring the disclosure cost, and being perceived as an average below-threshold firm while saving the cost of disclosure. In Verrecchia (1983) the underlying communication model is a monotonic signaling game (the firm always wants to be perceived having as high profits as possible), while in ours it is a Crawford-Sobel (non-monotonic) signaling game.[3] Therefore, his model can be seen as a special case of ours when the sender's bias is infinitely large and all but one of the free, unverifiable messages are disallowed. Naturally, in his model, the question whether a greater bias leads to more or less information transmission does not arise.

Other models with hard information transmission do not get full revelation by restricting the sender's (or senders') ability to prove what the state of nature is. Lipman and Seppi (1995) provide conditions on the sets of available messages under which it is feasible for one or more senders to identify the state. In Shin's (1994) more applied setting senders with opposing interests have hard evidence that the state lies above or below certain privately-known thresholds. In equilibrium, however, the senders suppress all

[2] Seidmann and Winter (1997) extend the result for more general environments.

[3] Another difference is that Verrecchia (1983) does not allow (free) unverifiable messages while we do.

information that is unfavorable for their own bias, and, interestingly, a more informed sender bears more of the burden of proof. In a recent paper Dziuda (2006) uses a structure similar to Shin's in a single-expert communication game. In her model not all unfavorable hard information gets suppressed, but full revelation is still impossible. Another model of communication with partial verifiability is that of Glazer and Rubinstein (2004, 2006). They study an environment where the sender can provide hard signal about only one of the two dimensions of the state of nature. The receiver has a binary action—accept or reject—which she would like to condition on the state. In contrast, the sender is commonly known to prefer one of the actions in all states.[4] Glazer and Rubinstein characterize equilibria and receiver-optimal communication mechanisms.

Our model is different from the above models of partial verifiability in that we expand the canonical model of cheap talk of Crawford and Sobel (1982) so that there is a direct (payoff) cost of sending a hard signal without altering any other aspect of the model, such as the state-dependence of the sender's preferences and the continuous nature of receiver's action.[5]

In the original Crawford-Sobel cheap talk model and most of its variants, better-aligned preferences lead to more communication and higher payoffs for the receiver. In an interesting recent paper Che and Kartik (2007) suggest a reason why and how this result may be overturned. They argue that getting advice (in the form of partially verifiable signals) from a more biased

[4]Note that as compared to the standard Crawford-Sobel cheap talk game, the state space here is more general (multidimensional), but the Receiver's action and the Sender's preferences are very much restricted (binary action, state-independent Sender preferences).

[5]A related strand of literature (see Ottaviani and Squintani (2006), Kartik, Ottaviani and Squintani (2007) and Kartik (2008)) introduces *costly lying* (or a credulous receiver, implying the same) in the Crawford-Sobel model. In these models the sender announces "his type", which is unverifiable but costly if he deviates from telling the truth. (In contrast, we assume that it is costly to report the truth in a verifiable way.) The structure of equilibria in these models are also quite different from that found in ours. For example, in the case of bounded state spaces, positive sender bias and lying costs, it is the low types of the sender that separate (by telling the truth) and the high types that pool (by reporting the same lie). In contrast, in our model with comparable preferences the high types separate by verifiably revealing their information and the low types pool by sending free, unverifiable messages.

expert may be beneficial for the decision maker because a biased sender may have a greater incentive to acquire such signals in order to persuade the receiver. This result is obtained under the assumption that the Sender and the Receiver have different opinions, i.e., that they do not share the same prior about the state of nature. Our model is set up and works differently from theirs. Instead of studying the costly *acquisition* of partially verifiable signals, we model the Sender's incentives for costly (and full) *verification* of already existing private information when cheap talk messages are also allowed. Perhaps the most important difference, however, is that in their model a difference of opinion (different priors) plays an important role, while we assume common priors.

Dewatripont and Tirole (2006) develop a moral hazard model of communication where the speaker and the listener *both* have to exert uncontractible effort in order to make the listener learn the state of nature. (In their base model the parties start symmetrically informed about the state and choose their effort levels simultaneously.) The receiver wants to condition his binary action on the state, while the sender wants to induce the same action irrespective of the state. Dewatripont and Tirole show that as the prior probability of the state where the receiver takes the sender's preferred action increases, the sender's incentive to exert effort decreases. They interpret this result as "an increase in congruence between S and R can lead to the breakdown of issue-relevant communication" (page 1226). Note, however, that in their model an "increase in congruence" does not mean that the parties' preferences become more aligned conditional on the state of nature. Instead, it means that the probability that getting information changes the receiver's action in the sender's favor decreases. In contrast, in our model, less communication can result from more aligned state-dependent preferences.

Though it does not feature hard information, Austen-Smith and Banks's (2000) model of burning money is related to our work. The structure of an equilibrium with burned money is similar to the structure of our equilibrium involving hard information. However, there are significant differences. Agents who burn money send costly and unverifiable ("cheap talk") messages, and they signal their type with the cost they incur. Our Sender types can send hard information at a fixed cost. In the Austen-Smith and Banks model there always are fully revealing equilibria, while in our model that is possible only if the cost of hard information is zero.

The paper is structured as follows. In Section 2 we set up a model of communication via cheap talk and costly hard (verifiable) signals. In Section 3 we prove that an interval-partition equilibrium exists. In Section 4 we show that in the uniform-quadratic case (frequently used in applications) our model implies that an increase in the sender's bias leads to more information transmission and a higher expected utility for the receiver. An extension (example) with a multidimensional state space is studied in Section 5. Section 6 concludes; omitted proofs are collected in an Appendix.

2 The model

There are two players, the Sender (he) and the Receiver (she). The Sender has private information about the payoff-relevant state of nature, which we represent by the realization of a one-dimensional random variable, θ, and call his *type*. We assume that the distribution of θ is continuous on a compact support normalized to $[0, 1]$. The Sender, having observed θ, sends a message m from a set $M \cup \{h_\theta\}$. Messages in M (a non-empty set) are interpreted as "soft" signals because any type of the Sender can send them. The message h_θ is a "hard" signal because it can only be sent by type θ, thus it identifies the Sender.[6] The Receiver observes m and picks an action, $y \in \mathbb{R}$.

We assume that each player's payoff is strictly concave in y and that for any given Sender type, each player has a finite most-preferred Receiver action. We denote the Sender's "ideal point" by $y^S(\theta)$, and the Receiver's by $y^R(\theta)$. Following Crawford and Sobel (1982), we also assume that y^S and y^R are both strictly increasing and continuously differentiable, and that the sign

[6] More generally, we could allow hard signals of the form $\{h_T\}_{T \subseteq [0,1]}$ that can only be sent by types $\theta \in T$. However a model with this richer structure is not required for obtaining our main results.

of the Sender's bias, $b(\theta) \equiv y^S(\theta) - y^R(\theta)$, is the same for all θ. Without loss of generality, we normalize the sign of the Sender's bias to be positive, that is, $y^S(\theta) > y^R(\theta)$ for all θ. In the classical cheap talk model without hard signals, these assumptions guarantee that in any equilibrium, the set of types sending a given message form an interval, and that only a finite number of soft signals are transmitted.[7]

The main difference between our model and the classical cheap talk setup is that we assume the Sender can send a *type-revealing hard signal*, h_θ, at a positive cost, c. In addition, we assume that the Sender's payoff depends on his type and the Receiver's action only via the *distance* between y and $y^S(\theta)$. Therefore, the Sender's payoff can be written as $U^S(y^S(\theta) - y) - \mathbf{1}_{m=h_\theta}c$, where U^S is symmetric about and maximized at zero, and $\mathbf{1}_X$ is the indicator function (it equals 1 if X is true and 0 otherwise). We further assume that U^S is strictly concave. Finally, we assume that the Sender's bias, $b(\theta) \equiv y^S(\theta) - y^R(\theta)$, is not only positive, but also *weakly increasing* in θ. We will see in Proposition 2 that under these additional assumptions, in any equilibrium of our model with costly hard signals, Sender types that send the same soft message form intervals.[8]

The following example illustrates that when costly hard signals are available and $b(\cdot)$ is decreasing, the set of types that send the same cheap talk message need not be connected.

[7]See Crawford and Sobel (1982). The key assumption that guarantees interval-partition equilibria is that the parties' ideal points are monotonic. Gordon (2007) shows that if the Sender's bias switches signs over the type space then the number of equilibrium messages may be infinite.

[8]This result can be established more generally under the assumptions that the Sender's gross utility, $U^S(y,\theta)$, is such that for all θ and $y = y^R(\theta)$, $U^S_{11}(y,\theta) + U^S_{12}(y,\theta) \geq 0$, and for all $y \geq y^R(\theta)$, $U^S_{122}(y,\theta,b) \leq 0$. The former condition says that the single-crossing effect is stronger than the concavity of the Sender's payoff in y at $y = y^R(\theta)$; the latter implies that the single-crossing effect is weakly decreasing in θ as long as $y^R(\theta) \leq y$.

We use our more restrictive specification of the Sender's utility for expositional purposes. The analysis of the model under these more general conditions is available from the authors upon request.

Example 1 Let the Sender's ideal point be

$$y^S(\theta) = \begin{cases} 0.25 & \text{for } \theta \in [0, 0.2], \\ \theta + 0.05 & \text{for } \theta \in [0.2, 1]. \end{cases}$$

Let the distribution of types be an $(\varepsilon, 1-\varepsilon)$ mixture of the uniform distribution on $[0,1]$ and a point-mass at 0.5, where ε is small. Let the Receiver's ideal point be $y^R(\theta) \equiv \theta$, and Sender's utility $-(\theta + b(\theta) - y)^2$, where y is the Receiver's action. Set the cost of the hard signal to $c = 0.05$.

Direct calculations reveal (see `bnonmonotone.xls`, available from the authors) that in this example, if a soft signal results in a response $\bar{y} = 0.5$ then the gross (before-cost) gain of the Sender from sending the hard signal is zero at $\theta = 0$, increases to 0.06 at $\theta = 0.2$, then decreases to -0.0025 at $\theta = 0.45$, and from there it increases again up to 0.3 at $\theta = 1$. At cost $c = 0.05$ the types that prefer to send the hard signal are in a neighborhood of 0.2 (≈ 0.14 to 0.22) and above ≈ 0.68. For ε sufficiently small, the expected value of θ conditional on not falling into these two ranges is still approximately 0.5. Therefore, for ε sufficiently small, we have an equilibrium where the set of types that send a soft signal (babble) belong to the union of two disjoint intervals, $[0, 0.14]$ and $[0.22, 0.68]$. ■

In the next section we show that in any equilibrium of our game the types that send the same soft message form an interval while the types that send the hard signal form a finite union of intervals. The latter types may belong to intervals at either end of the type space, or be wedged between types that send different soft messages.

Our most surprising result is that under certain conditions (for example, in the leading uniform-quadratic specification), an increase in the Sender's bias leads to an increase in the amount of information that can be transmitted in equilibrium. The possibility of sending costly hard information can reverse the standard comparative statics result; more disagreement between the Sender and the Receiver can be advantageous for the Receiver as it induces more evidence production.

3 Existence of partition equilibria

The following Lemma is the key in proving both the existence and the interval-structure of equilibria in our model. It concerns the properties of the sender's gross gain (as a function of his type) from sending the hard signal instead of a given cheap-talk message. We show that this gross gain is strictly decreasing on an interval of low types, negative for types around the "average" type sending the particular cheap-talk message, and positive for sufficiently high types.

Lemma 1 *Fix $\underline{\theta}, \bar{\theta} \in [0,1]$ with $\underline{\theta} < \bar{\theta}$. Let $\theta_1 \in (\underline{\theta}, \bar{\theta})$ and $y_1 = y^R(\theta_1)$. There exists $\theta_0 \in [\underline{\theta}, \theta_1)$ such that $U^S(y^S(\theta) - y^R(\theta)) - U^S(y^S(\theta) - y_1)$ is*

(i) strictly decreasing in θ for all $\theta \in [\underline{\theta}, \theta_0)$,

(ii) negative for all $\theta \in (\theta_0, \theta_1)$,

(iii) strictly increasing in θ for all $\theta \in (\theta_1, \bar{\theta}]$.

Proof. See the Appendix. ■

The shape of $U^S(y^S(\theta) - y^R(\theta)) - U^S(y^S(\theta) - y_1)$ is illustrated in Figure 1.

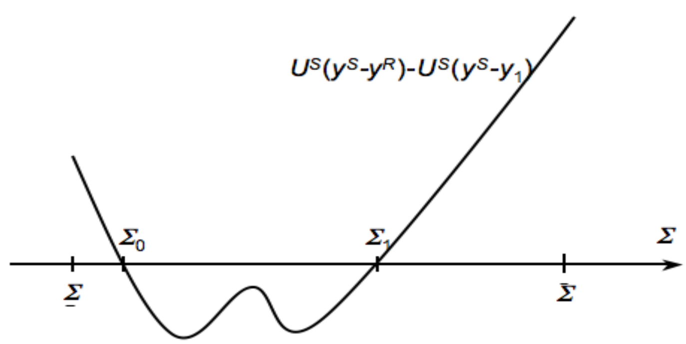

Figure 1: The gross gain from sending the hard signal

An immediate implication of Lemma 1 is that if there are any Sender types who prefer sending the hard signal at cost c compared to inducing $y_1 = y^R(\theta_1)$

via a soft (cheap-talk) message, then they are located at the extreme(s) of an interval containing θ_1. This is so because the net gain from sending h_θ over the message that results in y_1 is $U^S(y^S(\theta) - y^R(\theta)) - U^S(y^S(\theta) - y_1) - c$, which is decreasing for low θ's, increasing for high θ's and negative in the middle (for θ's near θ_1). To put it differently:

Remark 1 *Given any interval $[\underline{\theta}, \bar{\theta}] \subseteq [0,1]$ and action $y_1 = y^R(\theta_1)$ with $\theta_1 \in (\underline{\theta}, \bar{\theta})$, the Sender types in $[\underline{\theta}, \bar{\theta}]$ that weakly prefer action y_1 over sending the hard signal at cost c form a nonempty, closed interval around θ_1.*

For any interval $[\underline{\theta}, \bar{\theta}] \subseteq [0,1]$, let $\mu(\underline{\theta}, \bar{\theta}) = E\left[y^R(\theta) | \theta \in [\underline{\theta}, \bar{\theta}]\right]$. This is the Receiver's optimal action conditional on believing that the Sender's type belongs to the interval $[\underline{\theta}, \bar{\theta}]$.

Define

$$\bar{c} = \max_{\theta \in [0,1]} \left[U^S(y^S(\theta) - y^R(\theta)) - U^S(y^S(\theta) - \mu(0,1))\right].$$

This is the maximal utility gain a Sender type can obtain by sending the hard signal in a babbling equilibrium. By Lemma 1, the maximum is positive and is attained at either $\theta = 0$ or $\theta = 1$.

Clearly if $c > \bar{c}$, no hard information will be sent in any equilibrium. Our first proposition concerns the existence of equilibria when the cost of sending the hard signal is below the threshold $\bar{c}$.

Proposition 1 *If $c < \bar{c}$ then there exists an equilibrium where some types of the Sender send a hard signal. Moreover, there exists a cost $\underline{c}$ such that if $c < \underline{c}$, all equilibria involve hard information.*

Proof. For $\theta \in [0,1]$, let $S(\theta) \subseteq [0,1]$ denote the set of types that weakly prefer the outcome $y^R(\theta)$ to sending the hard signal at cost c.

Since the Sender with type θ (and types close by) strictly prefer the former, $S(\theta)$ is not empty. Indeed, it follows from Lemma 1 (see Remark 1) that $S(\theta)$ is an interval $[\underline{s}(\theta), \overline{s}(\theta)]$ with $0 \leq \underline{s}(\theta) < \theta < \overline{s}(\theta) \leq 1$.

Define the function $g : [0,1] \to [0,1]$ by

$$g(\theta) := \left(y^R\right)^{-1} \left(\mu(\underline{s}(\theta), \overline{s}(\theta))\right). \tag{1}$$

The functions $\underline{s}(\theta)$ and $\overline{s}(\theta)$ are continuous because the Sender's utility function is. μ is also continuous in its arguments because the cumulative distribution function F is differentiable. Finally, y^R is continuous and strictly increasing by assumption, hence the inverse exists and it is continuous. Thus g is well-defined and continuous. By Brouwer's Fixed Point Theorem, g has a fixed point θ^*. This determines an equilibrium where Sender types $\theta \in [\underline{s}(\theta^*), \overline{s}(\theta^*)]$ babble and induce $\mu(\underline{s}(\theta^*), \overline{s}(\theta^*)) = y^R(\theta^*)$, while the rest of the types send the hard signal.

Since $c < \bar{c}$, at least one type would strictly prefer sending a hard signal at cost c compared to the outcome of the babbling equilibrium, therefore hard information is indeed used in the equilibrium.

For the second statement, suppose there is an equilibrium that does not involve hard information. We know from Crawford and Sobel (1982) that the equilibrium has an interval partition structure, with the length of each interval bounded below by the minimum bias, $\underline{b} := \min_{\theta \in [0,1]} b(\theta)$. In particular, the interval containing type 1 is at least $\underline{b}$ long, and since the cumulative distribution function F is differentiable, the Receiver's response to this interval is strictly less than her response to type 1 revealing his type. Thus type 1 prefers to reveal his type; let $\underline{c}$ be the cost that makes him indifferent between that and the Receiver's response to the interval of length $\underline{b}$ including 1. As long as $c < \underline{c}$, type any equilibrium will have type 1 sending hard information. ■

Our next result is that all equilibria exhibit a certain "interval-partition property" familiar in cheap talk games. More precisely, we show that the set of types that send a given cheap talk message is connected, while the set of types that send a hard signal belong to a finite union of intervals.

Proposition 2 *In any equilibrium, there exist* $a_0 = 0 \leq a_1 \leq \ldots \leq a_N \leq a_{N+1} = 1$ *cutoff points such that types in the interval* (a_k, a_{k+1}) *either send* *the same cheap talk message, or all send a hard signal. Types belonging to different intervals do not send the same soft message.*

Proof. Since b is strictly positive, there exists $\varepsilon > 0$ such that $b(\theta) \geq \varepsilon$ for all $\theta \in [0, 1]$. Consider two actions $y < y'$ induced by cheap talk (soft) messages in an equilibrium. For types θ that choose y, $y^R(\theta) + b(\theta)$ is closer to y than to y'. For types θ' that choose y', $y^R(\theta') + b(\theta')$ is closer to y' than to y. Since b is continuously increasing, there must be a type m such that $y^R(m) + b(m)$ is half-way between y and y'. Moreover, all types below m must prefer y to y', and vice versa for types above m. Since the Receiver's ideal point at θ is $y^R(\theta)$, we have $y \leq y^R(m) \leq y'$. But $y^R(m) + b(m)$ is half-way between y and y', and $b(m) \geq \varepsilon$, so $y' - y \geq \varepsilon$. Thus there can only be finitely many cheap talk messages in any equilibrium.

The preceding argument also establishes that there cannot be three types $\theta < \theta' < \theta''$ such that θ and θ'' induce the same action through cheap talk, while θ' induces another action through cheap talk. By Lemma 1, no hard signal senders can be wedged in between types sending a given cheap talk message either. Therefore, types inducing any given action through cheap talk must form an interval, and there are finitely many such intervals. ■

Remark 2 *The proof of Proposition 2 also shows that if the bias $b(\cdot)$ is very large for all types, there exists no equilibrium in which more than one cheap-talk message is sent (i.e. "babbling" is the only equilibrium). But this means that if the cost c is not too high, an interval of types including 1 will prefer to send hard information. Thus for large biases and relatively small cost c, the unique equilibrium involves a cheap talk message sent by types below a cutoff type $\hat{\theta}$, and hard information sent by types above $\hat{\theta}$.*

The way equilibrium partitions are structured depends on the environment. In general, all we can say is that types sending the costly hard signal are located either at the extremes of the type space or in between types sending different soft messages.

In more specialized environments the structure of the equilibrium partition can be determined more precisely. For instance, suppose that

$$\mu(\theta,\theta') \leq \frac{y^R(\theta) + y^R(\theta')}{2} \text{ for all } \theta < \theta', \tag{2}$$

meaning that the Receiver's response if she believes the Sender's type is in $[\theta,\theta']$ does not exceed the *average* of her responses if she believes his type is either θ or θ', respectively.

Condition (2) is satisfied if $y^R(\theta) = \theta$ for all θ (the Receiver's ideal point is the state of nature) and the distribution of θ is concave (e.g., uniform). Under this assumption, all Sender types using the hard signal in equilibrium belong to the highest partition element, $[a_N, 1]$.

To see this assume towards contradiction that $[a_i, a_{i+1}]$ is an equilibrium partition-element and a_i is indifferent between sending the costly hard signal and a soft signal corresponding to $\theta \in [a_i, a_{i+1}]$. That is, suppose

$$U^S(b(a_i)) - c = U^S(\mu(a_i, a_{i+1}) - y^R(a_i) - b(a_i)),$$

where on both sides the argument of U^S is positive. By condition (2), $\mu(a_i, a_{i+1}) - y^R(a_i) \leq y^R(a_{i+1}) - \mu(a_i, a_{i+1})$, hence by the concavity of U^S,

$$U^S(b(a_i)) - c \geq U^S(y^R(a_{i+1}) - \mu(a_i, a_{i+1}) - b(a_i)).$$

Using $b(a_{i+1}) > b(a_i)$ and the strict concavity of U^S,

$$U^S(b(a_{i+1})) - c > U^S(y^R(a_{i+1}) - \mu(a_i, a_{i+1}) + b(a_{i+1})).$$

This means that Sender type a_{i+1} strictly prefers sending the hard signal to sending the soft message corresponding to $[a_i, a_{i+1}]$, which is a contradiction.

The strucure of equilibrium with hard signals when condition (2) holds is illustrated in Figure 2. There are two cheap talk messages sent in this equilibrium and types in $(a_2, 1]$ send hard signals (identify themselves at cost c).

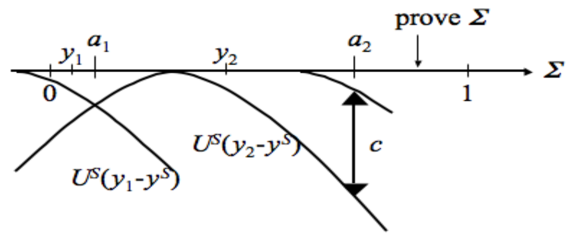

Figure 2: The structure of equilibrium in the uniform-quadratic case

The uniform-quadratic specification with a state-invariant bias is widely used in applications; for example, in political theory (see Gilligan and Krehbiel (1987, 1989)). When costly hard signals are available, this specification has other interesting properties as well. This is what we explore in the next section.

4 Disagreement and evidence production

A very intuitive and often cited feature of communication games is that if the parties' preferences diverge, less information is transmitted. The most interesting consequence of the availability of costly hard signals in our model is that a more severe disagreement between the Sender and the Receiver can yield *more* communication between the parties, and higher welfare for the Receiver. Of course, this phenomenon does not arise in every environment, but it can be observed in the leading specification of Crawford-Sobel cheap talk models, the uniform-quadratic case. In the following Proposition we claim that when the state is distributed uniformly on $[0, 1]$, the Receiver's ideal point is $y^R(\theta) \equiv \theta$ while the Sender's is $y^S(\theta) \equiv \theta + b$, with quadratic loss functions representing their preferences, an increase in b leads to an increase in information transmission and Receiver's welfare.

Before we state and prove the proposition it may be useful to discuss what "increased information transmission" exactly means and how it can be the result of a larger bias. Since the uniform-quadratic specification satisfies the condition (2), in an equilibrium with $(N+1)$ positive-length partition elements involving hard signals, the types that send the hard signal belong to the interval $[a_N, 1]$, as seen in Figure 2. The Sender with type a_N is indifferent between (1) revealing his type at cost c and incurring a loss b^2, and (2) sending the soft message corresponding to interval $[a_{N-1}, a_N]$ for free and incurring a loss of $(b + (a_N - a_{N-1})/2)^2$. As the bias increases, the increase in b^2 is smaller than the increase in $(b + (a_N - a_{N-1})/2)^2$ due to the convexity of the quadratic loss function. Therefore, *if the equilibrium partition remained the same* after b becomes larger, type a_N would strictly prefer to send the hard signal and the interval of types sending the hard signal would expand.

However, as b increases the cutoffs $a_1, \ldots, a_{N-1}$ determining the equilibrium partition do not remain the same. What the proof of Proposition 3 establishes is that as b goes up all cutoffs decrease, moreover, the distances between adjacent cutoffs decrease as well. Therefore, as b increases, each interval of types sending a given soft message shrinks, while the interval of types sending the hard signal expands. This represents a clear increase in information transmission, no matter how it is measured—in terms of entropy, the Receiver's payoff, etc. We conclude that a small increase in the bias almost always increases information transmission and the Receiver's welfare.[9]

Proposition 3 *Assume that θ is uniform on $[0,1]$, $y^R(\theta) \equiv \theta$, $y^S(\theta) \equiv \theta + b$ with $b > 0$, and both players have quadratic loss functions. Fix an equilibrium in which at least one Sender type sends hard information. An infinitesimal increase in b almost always increases the amount of information transmitted in equilibrium and the Receiver's expected payoff.*

Proof. See the Appendix. ■

[9] Note that if b increases substantially then the number of soft messages that can be transmitted in equilibrium will decrease. However, this is not the case for infinitesimal changes in b when the initial partition is non-degenerate (each interval has positive length). This explains why we say a small increase in b "almost always" increases information transmission.

5 Extension to multidimensional state spaces

If the Sender's information is multidimensional, information can be transmitted through cheap talk even if the preferences of the Sender and the Receiver are very different. In fact, if the Sender's bias is symmetric across the two states, then *comparative information* can always be transmitted in a cheap talk equilibrium regardless of how divergent preferences are (Chakraborty and Harbaugh (2006)). In the comparative cheap talk equilibrium the sender reports which coordinate of the state of nature is the largest, second largest, and so on. But even in this setup, the availability of costly hard information can change equilibria. We demonstrate through an example how hard information can be combined with cheap talk signals in a two-dimensional state space. Our main results hold in this example. First, the sender types that engage in hard information transmission are "extreme" (located in a corner of the state space). Second, an increase in the cost of hard information increases the informativeness of the equilibrium. The example also exhibits an interesting feature that is qualitatively different from the one-dimensional case: hard information sent about one dimension can, at the same time, give soft information about the other dimension.

Suppose the state, (ω_1, ω_2), belongs to $[0,1]^2$, and the prior distribution is uniform. The Receiver's ideal point is the state, the Sender's is (ω_1+b, ω_2+b), where $b > 0$. The utility of each player is the quadratic loss function, i.e. the negative of the square of the Euclidean distance between the action taken by the Receiver and the player's ideal point. This is a special case of the symmetric model of Chakraborty and Harbaugh (2006).

If there are no hard signals then there exists a two-message equilibrium with comparative information: the Sender reveals whether or not $\omega_1 \geq \omega_2$, and the Receiver replies with action $(2/3, 1/3)$ if $\omega_1 \geq \omega_2$ and action $(1/3, 2/3)$ otherwise. We analyze the effect of the availability of costly hard information on the two-partition "comparative cheap talk" equilibrium that Chakraborty and Harbaugh (2006) study.

Suppose that at cost $c > 0$ the Sender can prove either of the coordinates of ω. (Contrast this with Glazer and Rubinstein's persuasion game where exactly one coordinate can be proven for free, there are no soft messages, and the Receiver's action is binary.) What is the maximum level of c where hard information may be sent in equilibrium? In the two-partition equilibrium, type $\omega = (1, 1)$ has a loss of $(2/3+b)^2+(1/3+b)^2$. If he reports that $\omega_1 \geq \omega_2$ and reveals either coordinate then he obtains a loss of $2b^2$. He is indifferent between the comparative cheap talk equilibrium and revealing costly hard information if

$$c = \left(\frac{2}{3} + b\right)^2 + \left(\frac{1}{3} + b\right)^2 - 2b^2 = \frac{5}{9} + 2b.$$

As long as $c < 5/9 + 2b$, some Sender types will prefer sending costly hard information to the outcome of the comparative cheap talk equilibrium.

An equilibrium with hard information will look like the one depicted in Figure 3. In this equilibrium the Sender announces whether or not $\omega_1 \geq \omega_2$; moreover, if the state falls into either of the shaded areas, he verifies the largest coordinate. For example, if the state is $x = (x_1, x_2)$ as shown in the figure, the Sender reveals $\omega_1 \geq \omega_2$ and verifies $\omega_2 = x_2$. If the Sender does not verify a coordinate then the Receiver's reaction is either a or a' ; while if he does then the Receiver chooses the midpoint of the interval that is consistent with his report (e.g., in the case of state x or z , where ω_1 is the verified coordinate, the Receiver picks y).

In the rest of this section we argue that this is indeed an equilibrium when c is sufficiently close to $5/9 + 2b$. The reason we need large c is that the area where either coordinate is verified must be small relative to b, just like in the figure. We proceed in two steps.

First, we determine the set of types that send the hard signal. Restrict attention to states where $\omega_1 \geq \omega_2$. The states in which a hard signal is sent are those (x_1, x_2) that satisfy the inequality

$$-(x_1 + b - a_1)^2 - (x_2 + b - a_2)^2 \leq -(x_2 + b - y_2)^2 - b^2 - c. \tag{3}$$

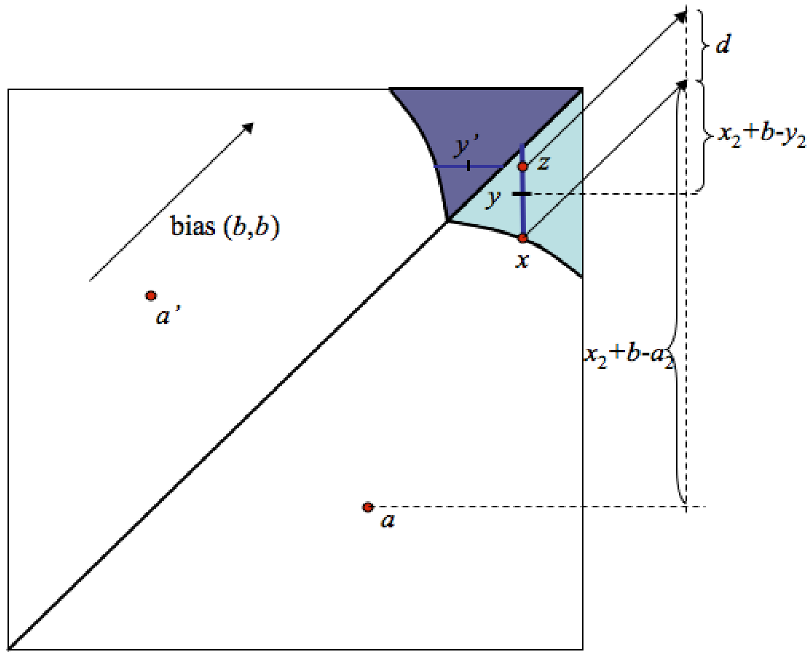

Figure 3: Types in the shaded areas send hard information

Here $y = (y_1, y_2)$ is the midpoint of the vertical line segment in $[0, 1]^2$ connecting $x = (x_1, x_2)$ and (x_1, x_1).

Note that $a = (a_1, a_2)$ is determined endogenously as the expected state when $\omega_1 \geq \omega_2$ and ω_1 is not verified. For each a (with $0 \leq a_1 \leq 1$ and $0 \leq a_2 \leq a_1$), let $S(a)$ be the subset of $\{x \in [0, 1]^2 | x_1 \leq x_2\}$ defined by the inequality (3). ($S(a)$ is empty for some a.) $S(a)$ is well-defined for every $a \in \{x \in [0, 1]^2 | x_1 \leq x_2\}$, and varies continuously with a. Thus the expectation map $f(a) = \mathbb{E}[x|S(a)]$ is also continuous, and so has a fixed point, which is the equilibrium value of a.

Second, we show that the types who send hard information form a connected region "in the corner," as shown in Figure 3. We claim that the Sender reports $\omega_1 \geq \omega_2$ and verifies ω_1 at all z such that $z_2 > x_2$ and $z_1 = x_1$. To see this, note that state z can be written as $(z_1, z_2) = (x_1, x_2 + d)$, with $d > 0$, hence

$$(z_1+b-a_1)^2+(z_2+b-a_2)^2 = (x_1+b-a_1)^2+(x_2+b-a_2)^2+2(x_2+b-a_2)d+d^2 \tag{4}$$

and

$$(z_2 + b - y_2)^2 + b^2 + c = (x_2 + b - y_2)^2 + b^2 + c + 2(x_2 + b - y_2)d + d^2. \tag{5}$$

As seen in the figure, $x_2+b-a_2 > x_2+b-y_2$ (because the hard-signal region is small), which combined with (4), (5) and the equality version of (3) yields

$$(z_1 + b - a_1)^2 + (z_2 + b - a_2)^2 > (z_2 + b - y_2)^2 + b^2 + c, \tag{6}$$

i.e. the Sender strictly prefers to announce $\omega_1 \geq \omega_2$ and verify ω_1 at z.

In the equilibrium described above, the hard signal sent by a type z serves, at the same time, as soft information: the Receiver learns from the hard signal z_1 that $z_2 \geq x_2$ (see Figure 3).

In this model, as the Sender's bias increases, the region in the state space where hard information is sent expands for the same reasons that it did in the uniform-quadratic specification of the one-dimensional model. Types on the boundary between the hard- and soft-information regions—characterized by (3) with an equality—strictly gain from verifying the highest coordinate of the state when b infinitesimally increases. This is so because in (3) the left-hand side decreases faster in b than the right-hand side does. This property of the one-dimensional model appears to generalize to multiple dimensions and comparative cheap talk.

6 Conclusion

We have studied the equilibrium "mixture" of evidence-based, costly hard signals and pure cheap talk in an information transmission game with bias. We have shown that the possibility of sending costly hard information in a Crawford–Sobel cheap talk game can be significant for the model's predictions. We have provided conditions under which all equilibria of the game are "interval-partition" equilibria. In the leading uniform-quadratic specification we have shown that hard information is sent by the Sender in the highest states of nature.

Most interestingly in the uniform-quadratic model, if hard information is sent in an equilibrium, an increase in the bias of the Sender leads to *more* information transmission. If hard information can be transmitted by the Sender at not too great a cost, then the Receiver might be better off choosing a more biased Sender. A legislature, for example, might be better of with a more biased committee making a proposal if the committee can transmit hard information such as expert testimony.

Much work remains to be done regarding the exact modeling of different forms of communication (*Logos*, *Ethos* and *Pathos* is just one classification), and very little is known about the tradeoffs that a speaker faces when choosing his arguments (or even his audience).

7 Appendix: Omitted Proofs

Proof of Lemma 1. If $y^S(\underline{\theta}) \geq (y_1 + y^R(\underline{\theta}))/2$ then let $\theta_0 = \underline{\theta}$. If $y^S(\underline{\theta}) < (y_1 + y^R(\underline{\theta}))/2$ then let $\theta_0 \in (\underline{\theta}, \theta_1)$ be the unique solution to $y^S(\theta_0) = (y_1 + y^R(\theta_0))/2$. Such a solution exists because $y^S(\theta)$ increases faster than $y^R(\theta)/2$ by the (weak) monotonicity of $b(\theta) \equiv y^S(\theta) - y^R(\theta)$, all involved functions are continuous, and $y^S(\theta_1) > y^R(\theta_1) = y_1$.

To see part (i), first note that for all $\theta \in [\underline{\theta}, \theta_0)$, $y^S(\theta) - y_1$ is negative and increasing, and so $U^S(y^S(\theta) - y_1)$ whose argument is negative is increasing as well. On the other hand, $U^S(y^S(\theta) - y^R(\theta))$ is decreasing in θ because $y^S(\theta) - y^R(\theta)$ is positive and increasing, and U^S is decreasing when its argument is positive. Therefore the difference $U^S(y^S(\theta) - y^R(\theta)) - U^S(y^S(\theta) - y_1)$ is decreasing in θ for $\theta \in [\underline{\theta}, \theta_0)$.

For all $\theta > \theta_0$, we have $y^S(\theta) > (y_1 + y^R(\theta))/2$ because the inequality holds weakly at $\theta = \theta_0$ by construction, and the left-hand side increases faster in θ than the right-hand side does. For all $\theta < \theta_1$, $y^R(\theta) < y_1$. Therefore, for all

$\theta \in (\theta_0, \theta_1)$, the Sender's ideal point is closer to y_1 than it is to $y^R(\theta)$, hence $U^S(y^S(\theta) - y^R(\theta)) < U^S(y^S(\theta) - y_1)$, and part (ii) follows.

Finally, we establish part (iii). The derivative of $U^S(y^S(\theta) - y^R(\theta)) - U^S(y^S(\theta) - y_1)$ with respect to θ is

$$\left[\dot{U}^S(y^S(\theta) - y^R(\theta)) - \dot{U}^S(y^S(\theta) - y_1)\right] \dot{y}^S(\theta) - \dot{U}^S(y^S(\theta) - y^R(\theta))\dot{y}^R(\theta),$$

where the symbol $\dot{}$ denotes the derivative of the corresponding function. If $\theta > \theta_1$ then $y^R(\theta) > y_1$, hence $y^S(\theta) - y^R(\theta) < y^S(\theta) - y_1$. Therefore, by the concavity of U^S the bracketed expression is positive. By assumption, $\dot{y}^S(\theta)$ and $\dot{y}^R(\theta)$ are positive. However, $\dot{U}^S(y^S(\theta) - y^R(\theta)) < 0$ because $y^S(\theta) > y^R(\theta)$. Therefore the derivative of $U^S(y^S(\theta)-y^R(\theta))-U^S(y^S(\theta)-y_1)$ with respect to θ is positive for $\theta > \theta_1$. ■

Proof of Proposition 3. Denote the interval-endpoints of the equilibrium partition by $a_0 = 0 < a_1 < \cdots < a_N < a_{N+1} = 1$. As we noted in the main text, for $k = 0, \ldots, N-1$, types in (a_k, a_{k+1}) send a cheap talk message and induce the action $\mu(a_k, a_{k+1})$, while $\theta \in (a_N, a_{N+1})$ produce hard evidence and induce the action θ.

For all $k = 1, \ldots, N-1$, type a_k is indifferent between pooling with types in (a_{k-1}, a_k) and those in (a_k, a_{k+1}). The corresponding arbitrage condition can be written as

$$-\left(a_k + b - \frac{a_{k-1} + a_k}{2}\right)^2 = -\left(\frac{a_k + a_{k+1}}{2} - a_k - b\right)^2.$$

The expressions inside the parentheses are positive, so we take square roots to get the second-order linear difference equation $a_{k+1} = 2a_k - a_{k-1} + 4b$ for $k = 1, \ldots, N-1$ with boundary condition $a_0 = 0$. The solution, parametrized by a_N, is given by

$$a_k = \frac{k}{N} a_N - 2k(N-k)b, \quad \text{for } k = 1, \ldots, N. \tag{7}$$

Type a_N is indifferent between sending the cheap talk message corresponding to (a_{N-1}, a_N) and the hard signal:

$$-b^2 - c = -\left(a_N + b - \frac{a_{N-1} + a_N}{2}\right)^2.$$

Using equation (7) for $k = N - 1$, this becomes

$$-b^2 - c = -\left(\frac{a_N}{2N} + Nb\right)^2.$$

Total differentiation with respect to b and a_N yields,

$$\frac{da_N}{db} = -2N^2 \frac{a_N + 2(N^2 - 1)b}{a_N + 2N^2 b}. \tag{8}$$

Note that an infinitesimal increase in b leads to a small decrease in a_N.

Differencing (7) for k and $(k - 1)$ yields

$$a_k - a_{k-1} = \frac{1}{N} a_N + 2b(2k - 1 - N).$$

Differentiating this in b and using (8) for da_N/db yields

$$\frac{d(a_k - a_{k-1})}{db} = -2N \frac{a_N + 2(N^2 - 1)b}{a_N + 2N^2 b} + 2(2k - 1 - N).$$

The first term is maximized (minimized in absolute value) at $a_N = 0$, while the last term is maximized at $k = N$. Therefore,

$$\frac{d(a_k - a_{k-1})}{db} < -2\frac{(N^2 - 1)}{N} + 2(N - 1) = 2(N - 1)\left(1 - \frac{N + 1}{N}\right) < 0.$$

We conclude that all soft-message intervals shrink as b increases. ■

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