

# Final Exam Study Guide for Calculus II

The final exam will be a 3 hour CUMULATIVE exam, and you are responsible for everything that we have covered this term. There will certainly be an emphasis on power series, Taylor series, and analytic functions, so make sure to focus on those topics. As a study aid, I have listed below the major definitions, results, proofs, techniques, and ideas that you are responsible for. I have NOT individually listed every differentiation / antidifferentiation formula that you should know (things like the derivatives / antiderivatives of logarithmic and exponential functions). In addition, you certainly need to know all of the basic techniques from Calculus I like the product rule, the chain rule, etc.

## Definitions:

1. The definite integral of a continuous function  $f$  on an interval  $[a, b]$
2. Improper integrals of type I and type II
3. Infinite sequence and convergence / divergence of infinite sequences
4. Infinite series and convergence / divergence of infinite series
5. Absolutely convergent and conditionally convergent
6. Power series centered at  $a \in \mathbb{R}$
7. Radius of convergence / interval of convergence of a power series. If you know the radius do you know the interval? If you know the interval do you know the radius?
8. Taylor series of a function  $f$  defined near  $a$  and infinitely differentiable at  $a$  (called the MacLaurin series if  $a = 0$ )

9. Taylor polynomials and Taylor remainders of a function  $f$  at  $a$
10.  $f$  is analytic at  $a$

**Results:**

1. Basic properties of the integral (pp. 373-5 of the text)
2. The Fundamental Theorem of Calculus (I and II)
3. Change of Variables (called “The Substitution Rule for Definite Integrals” in the text, p.404)
4. Monotonic Sequence Theorem
5. Divergence Test
6. Limit Laws for sequences and series
7. Integral Test
8. Comparison Test and Limit Comparison Test
9. Alternating Series Test
10. Absolute convergence implies convergence
11. Ratio Test
12. Root Test
13. Power series can be differentiated and integrated “term-by-term” inside their radius of convergence.
14.  $f$  is analytic at  $a$  *if and only if* there exists  $d > 0$  such that for all  $|x - a| < d$  we have  $\lim_{n \rightarrow \infty} R_n^a(f)(x) = 0$ .
15. Taylor’s Inequality

**Proofs:**

1. The Fundamental Theorem of Calculus (I and II)
2. Change of Variables

3. Comparison Test
4. Absolutely convergent implies convergent
5. Root Test

**Techniques:**

1. Techniques of Integration:
  - a) Substitution
  - b) Integration by Parts
  - c) Using trig identities to compute integrals of trig functions
  - d) Reverse (trigonometric) substitution
  - e) Partial Fractions
2. Find the radius of convergence of a power series by using the ratio test.
3. Find power series representations of functions by utilizing the geometric series and term-by-term integration / differentiation of power series.
4. Compute the Taylor / MacLaurin series of a function directly from the definition.
5. Use Taylor's Inequality to show that a function is analytic.
6. Compute the areas of interesting regions by integration.
7. Compute volumes by cylinders on cross-sectional slices or cylindrical shells ... especially solids of revolution.

**Ideas:**

1. Motivation for the definite integral in terms of the area problem: this should include a good picture or series of pictures and a short paragraph describing the approximation method. The goal is to convince me that you know *why* the definite integral is defined as a limit of Riemann sums.
2. Interpretation of the Fundamental Theorem of Calculus I and II as the statement that differentiation and integration are inverse processes. Describe *intuitively* why these results are true.

3. Why is an improper integral defined as the limit of definite integrals? Put otherwise: why does taking the limit of certain definite integrals make sense of our desire to find the area of an infinite region?
4. Why is an infinite series defined as the limit of the sequence of partial sums? Put otherwise: why does taking the limit of the partial sums make sense of our desire to “add up” infinitely many numbers?
5. List (with explanation) some reasons that analytic functions are “super-nice”.

**Basic Convergence / Divergence Results:** *These are your friends ... in combination with our various convergence tests, they will allow you to analyze the behavior of many sequences and series.*

1.  $\int_1^\infty \frac{1}{x^p}$  is convergent for  $p > 1$  and divergent for  $p \leq 1$ .
2. For which  $r \in \mathbb{R}$  does the sequence  $\{n^r\}$  converge and what is the limit?
3. For which  $r \in \mathbb{R}$  does the sequence  $\{r^n\}$  converge and what is the limit?
4. Geometric Series Theorem
5. Convergence / divergence of  $p$ -series
6. Divergence of the harmonic series

**Complex Numbers:**

1. Geometric picture of addition and multiplication
2. Cartesian form vs. polar form
3. De Moivre’s Theorem and its relationship to trigonometric identities