Midterm II Study Guide for Calculus II

Definitions: *Be able to state these precisely and explain what they mean.*

1. Improper integrals of type I and type II
2. Infinite sequence and convergence / divergence of infinite sequences
3. Monotonic sequence
4. Bounded sequence
5. Infinite series and convergence / divergence of infinite series
6. Alternating series
7. Absolutely convergent and conditionally convergent

Results: *Be able to state these precisely, including all hypotheses, and be able to use them to investigate the convergence / divergence of sequences and series.*

1. Monotonic Sequence Theorem
2. Divergence Test
3. Limit Laws for sequences and series
4. Integral Test
5. Comparison Test
6. Limit Comparison Test
7. Alternating Series Test
8. Absolute convergence implies convergence
9. Ratio Test
10. Root Test

Proofs: *Be able to give complete and careful proofs of these results.*

1. Comparison Test
2. Absolutely convergent implies convergent
3. Root Test

Techniques:

1. Use a partial sum to approximate an infinite series, and estimate the error by means of improper integrals.
2. Use a partial sum to approximate an alternating series, and estimate the error by means of the following fact: the error is bounded by the first omitted term.

Ideas: *Be able to write a short paragraph giving an intuitive answer for each.*

1. Why is an improper integral defined as the limit of definite integrals? Put otherwise: why does taking the limit of certain definite integrals make sense of our desire to find the area of an infinite region?
2. Why is an infinite series defined as the limit of the sequence of partial sums? Put otherwise: why does taking the limit of the partial sums make sense of our desire to "add up" infinitely many numbers?

Basic Convergence / Divergence Results: *These are your friends ... in combination with our various convergence tests, they will allow you to analyze the behavior of many sequences and series.*

1. $\int_1^\infty \frac{1}{x^p}$ is convergent for $p > 1$ and divergent for $p \leq 1$.
2. For which $r \in \mathbb{R}$ does the sequence $\{n^r\}$ converge and what is the limit?
3. For which $r \in \mathbb{R}$ does the sequence $\{r^n\}$ converge and what is the limit?
4. Geometric Series Theorem
5. Convergence / divergence of p-series
6. Divergence of the harmonic series

Complex Numbers:

1. Geometric picture of addition and multiplication
2. Cartesian form vs. polar form
3. De Moivre's Theorem and its relationship to trigonometric identities