PHYS160 SOLUTIONS Spring 2009

**Problem Set #7: Due in class on Fri. 6/5**

**Problems from Chapter 13 of Thornton & Rex:** 8, 14, 26, 38

**Problems from Chapter 14 of Thornton & Rex:** 1, 8, 13, 14, 17

13-8 Calculate the ground state Q values for the following reactions. Are the reactions endothermic or exothermic?

(a) $^{16}O(d,\alpha)^{14}N$ or... $^{16}O + {}^{2}H \rightarrow {}^{14}N + {}^{4}He$

Use Eq. 13.7 $Q = M_{O16}c^2 + M_{H2}c^2 - M_{N14}c^2 - M_{He4}c^2$

$M_{O16} = 15.994915u$ $M_{H2} = 2.014102\,u$

$M_{N14} = 14.003074u$ $M_{He4} = 4.002603u$

$Q = 0.00334u \times 931.49\ MeV/_{c^2 \cdot u} \cdot c^2 = \boxed{+3.111\ MeV}$ exothermic

(b) $^{12}C(^{12}C,d)^{22}Na$ or $^{12}C + {}^{12}C \rightarrow {}^{22}Na + {}^{2}H$

$Q = 2M_{C12}c^2 - M_{Na22}c^2 - M_{H2}c^2$

$M_{C12} = 12.000\,u$ $M_{Na22} = 21.994437u$ $M_{H2} = 2.014102u$

$Q = -0.008539 \times 931.49\ MeV = \boxed{-7.954\ MeV}$

endothermic

(c) $^{23}Na(p,{}^{12}C)^{12}C$ or... $^{23}Na + {}^{1}H \rightarrow {}^{12}C + {}^{12}C$

$M_{Na23} = 22.989770\,u$ $M_{H1} = 1.007825u$ $M_{C12} = 12.000u$

$Q = M_{Na23}c^2 + M_{H1}c^2 - 2M_{C12}c^2$.

$Q = -0.002405u \times 931.49\ MeV/u = \boxed{-2.240\ MeV}$

endothermic

**8-14** A slow neutron is absorbed by $^{10}B$ in the reaction $^{10}B(n,\gamma)^{11}B$. What is the energy of the $\gamma$ ray?

The reaction energy Q is... $Q = M_{B10}c^2 + M_nc^2 - M_{B11}c^2$

$M_{B10} = 10.012937\,u$ $M_{B11} = 11.009305$ $M_n = 1.008665u$

$Q = +0.012297u \times 931.49\,MeV/u =$ $\boxed{11.45\ MeV}$

↑ The $\gamma$-ray will have slightly less energy than this as the $^{11}B$ nucleus will recoil.

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To estimate the Dopper shift of the $\gamma$-ray due to recoil of the $^{11}B$ nucleus, calculate the momentum of the photon...

$$P_\gamma = \frac{E_\gamma}{c} \cong 11.45\,MeV/c$$

To conserve momentum the $^{11}B$ nucleus must have the same momentum in the opposite direction. The kinetic energy of the $^{11}B$ nucleus is therefore,

$$K = \frac{P_\gamma^2}{2M}\left(\frac{c^2}{c^2}\right).$$

We can use non-relativistic equations since $Q << Mc^2$

$M_{B11}c^2 = 11.009305\,u \times 931.49\,\frac{MeV}{c^2} = 10255\,MeV$

less 5 electrons worth... $\underline{10{,}253\,MeV}$

$$K = \frac{(11.45\,MeV)^2}{2 \cdot 10253\,MeV} = 0.0064\,MeV$$

↑ negligible compared to Q

So..

$$\boxed{E_\gamma = 11.45\,MeV}$$

18-26 Calculate how much energy is released when $^{239}Pu$ absorbs a thermal neutron and fissions in the reaction

$$n + {}^{239}Pu \rightarrow {}^{240}Pu^* \rightarrow {}^{95}Zr + {}^{142}Xe + 3n$$

$$Q = M_{Pu239}c^2 + M_n c^2 - M_{Zr95}c^2 - M_{Xe142}c^2 - 3M_n c^2$$

combine ... $-2M_n c^2$

$M_{Pu239} = 239.052156\,u$ $\quad M_{Zr95} = 94.908043\,u$

$M_n = 1.008665\,u$ $\quad M_{Xe142} = 141.929710\,u$

$$Q = 0.197073\,u \times 931.49\,\text{MeV}/u = \boxed{183.6\,\text{MeV}}$$

13-38 Assume that 2/3 of the earth's surface is covered with water to an average depth of 3 km. Calculate how many nuclei of $^2H$ exist (0.015% abundance). Estimate how many joules of energy this represents?

First estimate the total mass of $H_2O$:

$$M = \text{density} \times \text{volume} = 1\frac{g}{cm^3} \times \frac{2}{3} \cdot 4\pi R_\oplus^2 \times \text{depth}$$

$$R_\oplus \simeq 6.4\times10^3\ m = 6.4\times10^5\ cm$$

$$\text{depth} = 3\ km = 3\times10^5\ cm$$

$$M = \frac{8}{3}\pi\left(6.4\times10^5\ cm\right)^2 \cdot 3\times10^5\ cm \cdot 1\frac{g}{cm^3}$$

$$\boxed{M \cong 10^{18}\ g}$$

The molar mass of $H_2O$ is ~ $18\frac{g}{mol}$ so there are

$$N_{H_2O} \simeq \frac{10^{18}\ g}{18\ g/mol} \sim 5\times10^{16} \text{ moles of } H_2O$$

or about...

$$5\times10^{16} \times 2 \times N_A \cong 7\times10^{40} \text{ hydrogen atoms}$$

↑ Avogadro's number $6.02\times10^{23}$ atoms/mole

Multiply by the % abundance... (÷100) to get the number of deuterons...

$$\boxed{N_{\text{deuterons}} \simeq 10^{37}}$$

The reactions listed in Eqs. 13.21 and 13.22 require 2 deuterons each... and result in an average energy release of 3.65 MeV, so the total energy potential of all the deuterium on earth is...

$$\frac{N_{\text{deuterons}}}{2} \times 3.65\,MeV \cong 2\times10^{37}\ MeV \times 10^6\frac{eV}{MeV} \times 1.6\times10^{-19}\,J$$

$$\boxed{E_{Total} = 3\times10^{24}\ J}$$

The current annual global energy demand is $5\times10^{20}$ J (wikipedia)

So that gives us ~ 6000 years of fuel at current rates of consumption.

**14-1** What are the frequencies of the two photons produced when a proton and an anti-proton annihilate?

$$p + \bar{p} \rightarrow \gamma + \gamma$$

If the proton and antiproton are at rest then each photon has an energy equal to the rest mass energy of the proton

$$E_\gamma = 938.27 \text{ MeV}$$

The frequency is...

$$f = \frac{E}{h} = \frac{938.27 \text{ MeV} \cdot 10^6 \text{ eV/MeV}}{4.1357 \times 10^{-15} \text{ eV} \cdot \text{s}}$$

$$\boxed{f = 2.27 \times 10^{17} \text{ Hz}}$$

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**14-8** Supply the missing neutrinos in the following reactions or decays.

(a) $\mu^+ \rightarrow e^+ + \underline{\ ?\ }$

To conserve $L_e$ and $L_\mu$ this decay requires 2 neutrinos

$$\boxed{\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu}$$

| | | | | | |
|---|---|---|---|---|---|
| $L_e$ | 0 | = | -1 | +1 | 0 |
| $L_\mu$ | -1 | = | 0 | 0 | -1 |

(b) $\underline{\ ?\ } + p \rightarrow n + e^+$ to conserve $L_e$, need $\bar{\nu}_e$ on left side.

$$\boxed{\bar{\nu}_e + p \rightarrow n + e^+}$$

| | | | | | |
|---|---|---|---|---|---|
| $L_e$ | -1 | 0 | = | 0 | -1 |

(c) $\pi^- \rightarrow \mu^- + \underline{\ ?\ }$ to conserve $L_\mu$, need $\bar{\nu}_\mu$ on the right side..

$$\boxed{\pi^- \rightarrow \mu^- + \bar{\nu}_\mu}$$

(d) $K^- \rightarrow \mu^- + \underline{\ ?\ }$ same as (c)... $\boxed{K^- \rightarrow \mu^- + \bar{\nu}_\mu}$

(e) $? + n \rightarrow p + \mu^-$ $\boxed{\nu_\mu + n \rightarrow p + \mu^-}$

**14-13** Explain why each of the following reactions is forbidden.

(a) $p + p \rightarrow p + p + n$

$B = +1 \quad +1 \quad \neq +1 \quad +1 \quad +1$

$S \quad \underbrace{\tfrac{1}{2} \quad \tfrac{1}{2}}_{=0,1} \qquad \underbrace{\tfrac{1}{2} \quad \tfrac{1}{2} \quad \tfrac{1}{2}}_{=\frac{1}{2},\frac{3}{2}} \leftarrow$

This does not conserve baryon number or spin/angular momentum

(b) $p + p \rightarrow p + \pi^+ + \gamma$

$B = +1 \quad +1 \quad \neq +1 \quad 0 \quad 0$

$S \quad \underbrace{\tfrac{1}{2} \quad \tfrac{1}{2}}_{0,1} \qquad \underbrace{\tfrac{1}{2} \quad 0 \quad 1}_{\frac{1}{2},\frac{3}{2}}$

This reaction does not conserve baryon number ~~or angular mo~~ or angular momentum

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**14-14** Explain why each of the following reactions is forbidden.

(a) $p + \bar{p} \rightarrow \mu^+ + e^-$ This does not conserve lepton numbers $L_\mu, L_e$

(b) $\gamma + p \rightarrow n + \pi^0$ This does not conserve charge or ~~angular momentum~~

$Q \quad 0 \quad +1 \quad \neq 0 \quad 0$

(c) $\gamma \rightarrow \pi^+ + \pi^-$ This does not conserve angular momentum

$S \quad 1 \neq \underbrace{0 \quad 0}_{0}$

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**14-17** Complete the following reactions:

(a) $\mu^- + p \rightarrow n + \nu_\mu$

(b) $n + p \rightarrow \Sigma^0 + n + K^+$

need (+) charge and $S = +1$, $B = 0$ to complete this reaction.