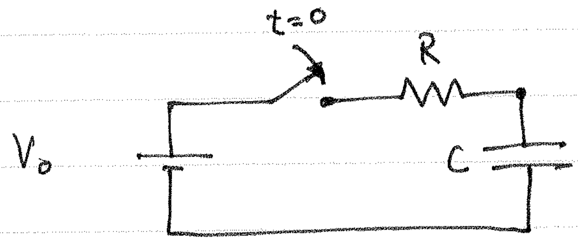
26.83

A capacitor is initially uncharged. It is in series with a resistor and an emf soure ($V_o = 110V$).



The time constant is

$\tau = RC = 6.2s$

And the initial current is

$I_o = 6.5 \times 10^{-5}$ A

What are the resistance and the capacitance?

The solution to this problem was developed in lecture.

$$I = I_o e^{-t/RC}$$

Current decays exponentially with time constant $\tau = RC$

Initially the voltage across the capacitor is $V_c = 0$ since $Q = 0$ at $t=0$.
Therefore the voltage across the resistor is $V_r = V_o = I_o R$ at $t=0$ by Kirchoff's loop rule.

So.. $$R = \frac{V_o}{I_o} = \frac{110V}{6.5 \times 10^{-5} A} = \boxed{1.7\ M\Omega}$$

From the time constant $\tau = RC = 6.2s$

$$C = \frac{\tau}{R} = \frac{6.2s}{1.7 \times 10^{6} \Omega} = \boxed{3.7\ \mu F}$$

$\boxed{26.86}$ An RC circuit has a time constant RC.

(a) If the circuit is discharging, how long will it take for its stored energy to be reduced to $1/e$ of its initial value?

The charge on the capacitor decays exponentially. $Q = Q_0 e^{-t/RC}$

But the stored energy is $W = \frac{1}{2}\frac{Q^2}{C} = \underbrace{\frac{Q_0^2}{2C}}_{W_0} e^{-2t/RC}$

initial or maximum stored energy

This decays with a time constant $\frac{RC}{2}$ that is half that of the charge.

$$\boxed{\tau = \frac{RC}{2}}$$

(b) If it is charging, how long will it take the stored energy to reach $1/e$ of its max value?

During charging, the charge on the capacitor is $Q = Q_0\left(1 - e^{-t/RC}\right)$

so the stored energy is $W = \frac{Q^2}{2C} = \underbrace{\frac{Q_0^2}{2C}}\left(1 - e^{-t/RC}\right)^2$

This is W_0, the maximum stored energy

Find the time when $W = \frac{W_0}{e}$ or... $e^{-1} = \left(1 - e^{-t/RC}\right)^2$

Take $\sqrt{\ }$ of both sides... $e^{-1/2} = 1 - e^{-t/RC}$ solve for t...

$$e^{-t/RC} = \left(1 - e^{-1/2}\right) \Rightarrow t/RC = -\ln\left(1 - e^{-1/2}\right)$$

or

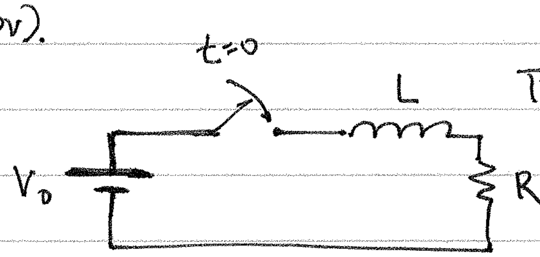
$$= -\ln\left(1 - \frac{1}{\sqrt{e}}\right) = \ln\left(\frac{1}{1 - \frac{1}{\sqrt{e}}}\right)$$

The stored energy reaches $1/e$ of its max value in a time slightly less than the RC time constant.

$$\boxed{\frac{t}{RC} = 0.93 \approx 1}$$

30.19

An inductor (2.50 H) with resistance of 8.00 Ω connected to an ideal battery (6.00V).



This is the effective circuit.

(a) Find the initial rate of increase of current in the circuit

At $t=0$, $I=0$, so there is no voltage drop across the resistor.

Kirchoff's loop rule therefore demands that ... $V_0 = L\frac{dI}{dt}$

or.. $\frac{dI}{dt} = \frac{V_0}{L} = \frac{6.00V}{2.50H} = \boxed{2.4\ A/s}$

(b) Find the rate of increase of current when $I = 0.500A$

At an arbitrary time Kirchoff's loop rule says that $V_0 - L\frac{dI}{dt} - IR = 0$

Solve for dI/dt ... $\frac{dI}{dt} = \frac{V_0}{L} - \frac{R}{L}\cdot I = \frac{6.00V}{2.50H} - \frac{8.00\Omega}{2.50H}\cdot 0.500A$

$= 2.4\,A/s - 1.6\,A/s$

$\boxed{\frac{dI}{dt} = 0.80\,A/s}$

(c) Find the current 0.250 s after the circuit is closed.

The solution to the differential equation that arises from Kirchoff's loop rule: $V_0 - L\frac{dI}{dt} - IR = 0$ is ...

$I = \frac{V_0}{R}\left(1 - e^{-Rt/L}\right)$ ... the time constant is $\tau = L/R$ for exponential approach to steady current.

So... at $t = 0.250\,s$... $Rt/L = t/\tau = \frac{8.00\,\Omega \cdot 0.250\,s}{2.50\,H} = \underline{\underline{0.8}}$

So.. $I = \frac{6.00\,V}{8.00\,\Omega}\left(1 - e^{-0.8}\right) = \boxed{0.413\,A}$

(b) The final, steady current is $I_0 = \frac{V_0}{R}$ because the current is no longer changing... there is no voltage drop across the inductor.

$$I_0 = \frac{V_0}{R} = \frac{6.00\,V}{8.00\,\Omega} = \boxed{0.750\,A}$$

$\boxed{30.32}$ A radio tuning circuit : Variable capacitor $\overset{C_{min}}{4.18pF} < C < C_{max}$

The circuit must tune ~~from~~ (resonate) from ~~1600 kHz < f <~~ $\underset{f_{min}}{540\ kHz} < f < \underset{f_{max}}{1600\ kHz}$

(a) What is the inductance of the coil in the tuning circuit?

The resonant frequency is $f_o = \frac{1}{2\pi\sqrt{LC}}$

When $C = C_{min} = 4.18\times10^{-12} F$ and $f_o = f_{max} = 1.6\times10^{6} Hz$,

then... solve for L..

$$(2\pi f_{max})^2 = \frac{1}{LC_{min}} \quad \text{or..} \quad L = \frac{1}{C_{min}\cdot(2\pi f_{max})^2}$$

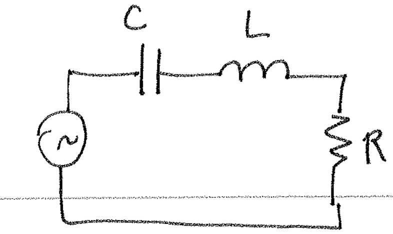
$$L = \frac{\cancel{10^{12}}}{4.18\cdot 4\pi^2\cdot(1.6)^2\cdot \cancel{10^{12}}} = 2.37\times10^{-3} H = \boxed{2.37 mH}$$

(b) Find C_{max}, using the resonant frequency at the other end of the AM band... f_{min}

$$C_{max} = \frac{1}{L(2\pi f_{min})^2} = \frac{10^3}{2.37}\cdot\frac{1}{4\pi^2\cdot(5.4)^2\cdot 10^{10}}$$

$$= \frac{10^{-7}}{2.37\cdot 4\pi^2\cdot(5.4)^2} = 3.67\times10^{-11} F = \boxed{36.7 pF}$$

$\boxed{31.21}$ Series LRC Circuit :



$V_0 = 3.00\,V$ amplitude
$C = 5.00\,\mu F$
$L = 0.400\,H$
$R = 200\,\Omega$

(a) At what frequency will the current be the greatest? What will be the current amplitude at this frequency?

The current will be greatest at the resonant frequency of $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.4\cdot 5\times 10^{-6}}}$

$= \frac{10^3}{2\pi\sqrt{2}} = \boxed{113\,Hz}$

or $\omega = 2\pi f = 707\,rad/s$

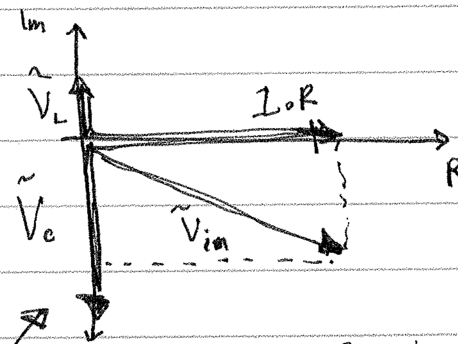
At this frequency the phasors for the voltage across the capacitor and the voltage across the inductor are equal in magnitude and cancel each other out, so... the voltages across the resistor must equal the voltage across the source...

$I_0 R = V_0$ or... $I_0 = \frac{V_0}{R} = \frac{3\,V}{200\,\Omega}$

$\cancel{\boxed{I_0 = 1.50\,mA}}$ $\boxed{I_0 = 15.0\,mA}$

(b) What will be the current amplitude when $\omega = 400\,rad/s$

Use phasors to solve this problem. Since $\omega < \omega_0$ the voltage across the capacitor will be larger than the voltage across the inductor



$V_{in} = \sqrt{(I_0 R)^2 + \left(-\frac{I_0}{\omega C} + \omega L I_0\right)^2}$

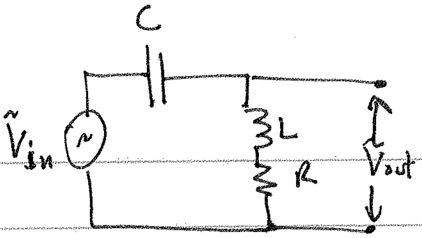
or... $I_0 = \frac{V_{in}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

$I_0 = \frac{3.00\,V}{\sqrt{(200\,\Omega)^2 + \left(400\cdot 0.4 - \frac{1}{400\cdot 5\times 10^{-6}}\right)^2}} = \frac{3.00}{\sqrt{155600}} = \boxed{7.61\,mA}$

From phasor diagram the source will lag behind the current.

31.49

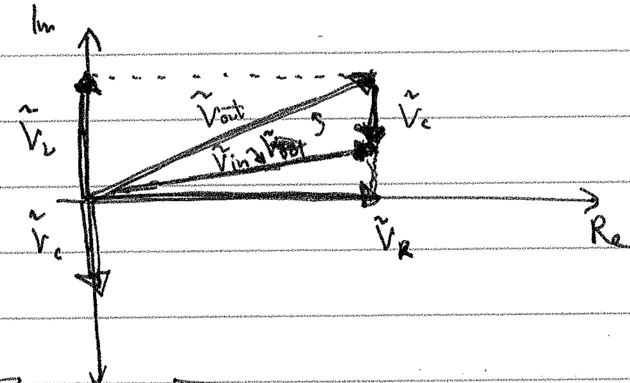
LRC High Pass Filter:



The output is taken across the real inductor with resistance R.

Show that this is a high pass filter...

Here is the phasor diagram:



$$V_{out} = \sqrt{(I_0 R)^2 + (I_0 \omega L)^2} = I_0 \sqrt{R^2 + (\omega L)^2}$$

$$V_{in} = \sqrt{(I_0 R)^2 + \left(I_0 \omega L - \frac{I_0}{\omega C}\right)^2} = I_0 \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

So.. $$\frac{V_{out}}{V_{in}} = \frac{\sqrt{R^2 + (\omega L)^2} \cdot \frac{1}{R}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \cdot \frac{1}{R}} = \frac{\sqrt{1 + \left(\omega \frac{L}{R}\right)^2}}{\sqrt{1 + \left(\omega \frac{L}{R} - \frac{1}{\omega RC}\right)^2}}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{\sqrt{1 + \left(\omega \frac{L}{R}\right)^2}}{\sqrt{1 + \left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)^2}}}$$

At low frequency... $$\frac{V_{out}}{V_{in}} \approx \frac{1}{\sqrt{1 + \frac{1}{\omega^2 (RC)^2}}} \sim \frac{1}{\sqrt{\frac{1}{\omega^2 (RC)^2}}} \approx \underline{\underline{\omega RC}}$$

linear with frequency.

At high frequency... $$\frac{V_{out}}{V_{in}} \approx \frac{\sqrt{\left(\frac{\omega L}{R}\right)^2}}{\sqrt{\left(\omega L/R\right)^2}} \approx \underline{\underline{1}}$$