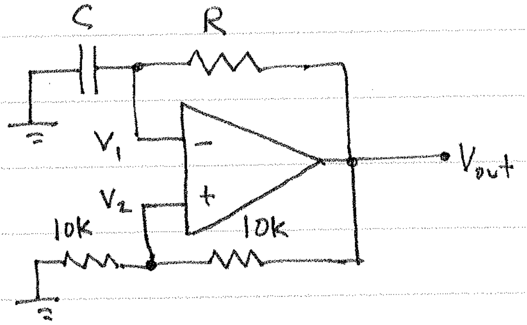
PHYS 220 Physical Electronics Fall 2010

Notes on Clocks and Oscillators

- Relaxation Oscillator using an Op·Amp as a "comparator" or "Schmitt trigger". See Horowitz and Hill, p. 285.



Output of op·amp will swing from $+V_{cc}$ to $-V_{cc}$ depending on whether $V_2 > V_1$ or $V_2 < V_1$.

Circuit has an unstable equilibrium: $V_1 = V_2 = V_{out} = 0$ no charge on C.

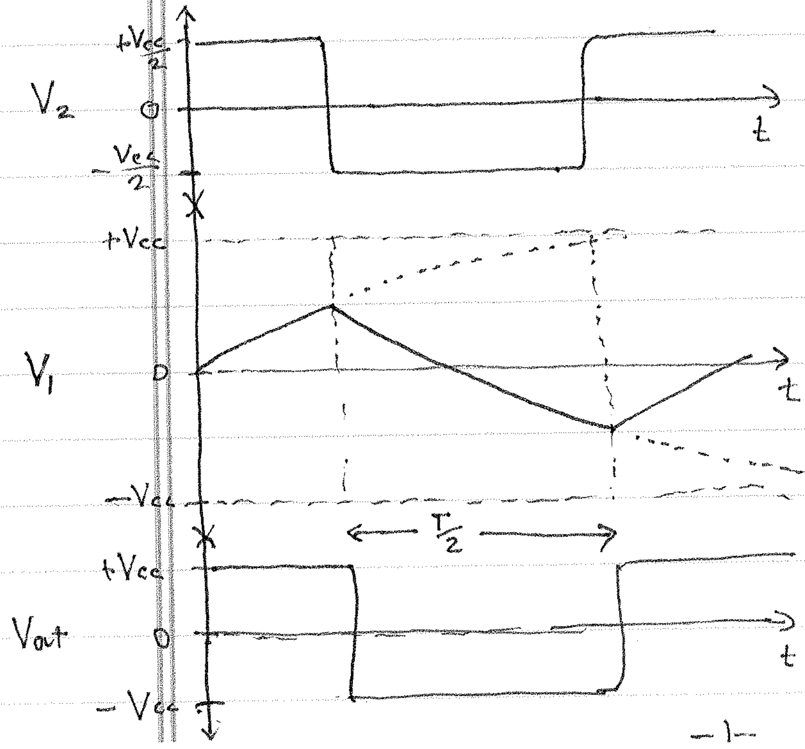
Consider a small perturbation ... $V_2 \rightarrow +\epsilon$

Because of large open-loop gain $V_{out} \rightarrow +V_{cc}$ and $V_2 \rightarrow \frac{+V_{cc}}{2}$

because of the voltage divider with 10k resistors.

Current flows through negative feedback network, charging capacitor.

V_1 increases on RC timescale until $V_1 \gtrsim +\frac{V_{cc}}{2}$ when the op-amp comparator swings its output to $-V_{cc}$ ...



In time $\frac{T}{2}$ (half the "clock" period)

V_1 decays toward $-V_{cc}$ from $+\frac{V_{cc}}{2}$ to $-\frac{V_{cc}}{2}$

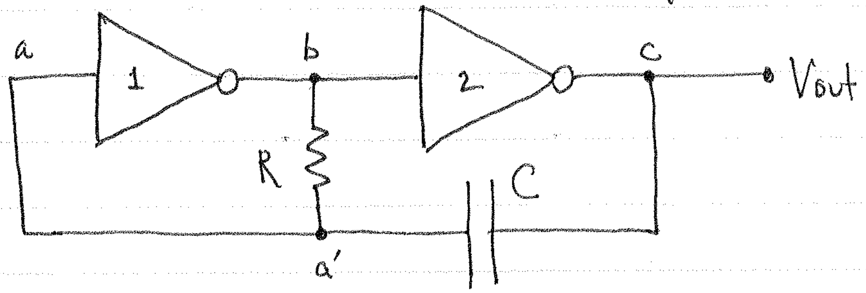
... ending up at $\frac{1}{3}$ of its initial voltage difference with respect to $-V_{cc}$.

So... $e^{-\frac{T}{2RC}} = \frac{1}{3}$

or $T = (2\ln 3)RC$

$$\boxed{T \doteq 2.2\,RC}$$

- Relaxation Oscillator using Inverter Gates (see Horowitz and Hill, p. 285).
  → "Inverter Clock"

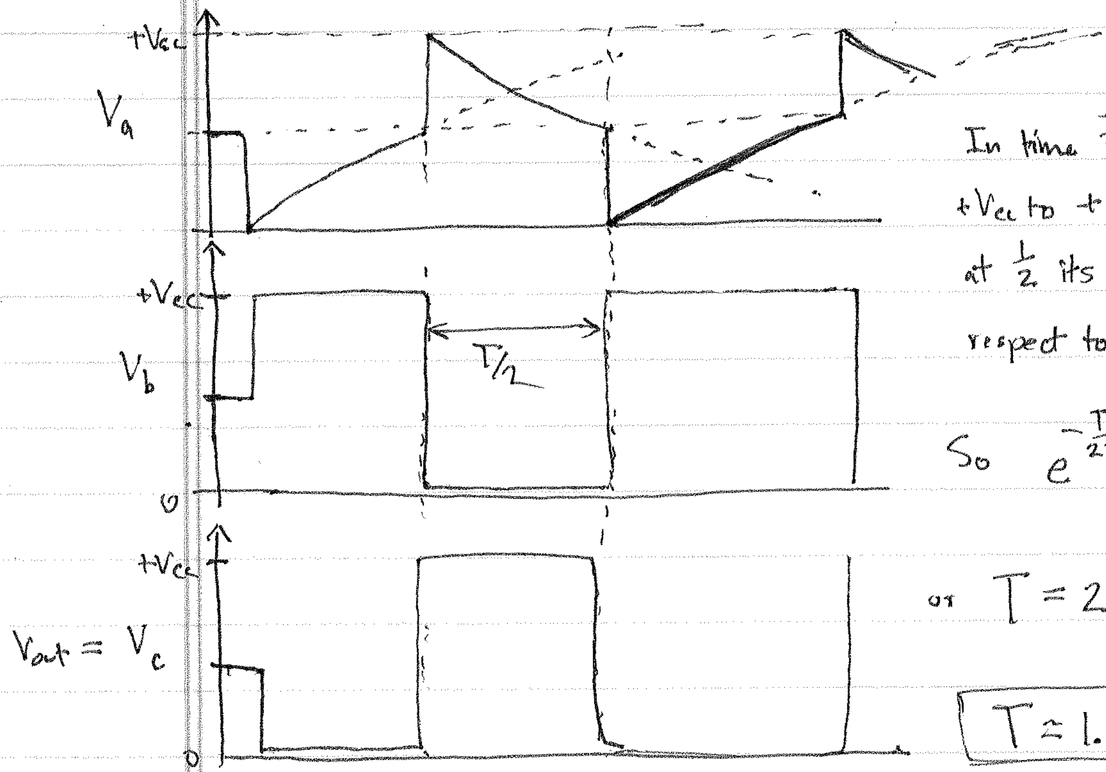


(1) This circuit has an unstable equilibrium at $V_a = V_b = V_c = \frac{V_{cc}}{2}$.

(2) Consider a small perturbation at b such that $V_b = \frac{V_{cc}}{2} + \epsilon$

(3) Inverter #2 output will swing LO. The initial low going pulse will couple through Capacitor C, pulling V_a LO and inverter #1 output will swing HI reinforcing the perturbation.

(4) Current will flow from b to a' and charge the capacitor. When $V_a = V_{a'} \approx \frac{V_{cc}}{2}$ Inverter #1 output will swing LO, inverter #2 output will swing HI, pulling V_a HI



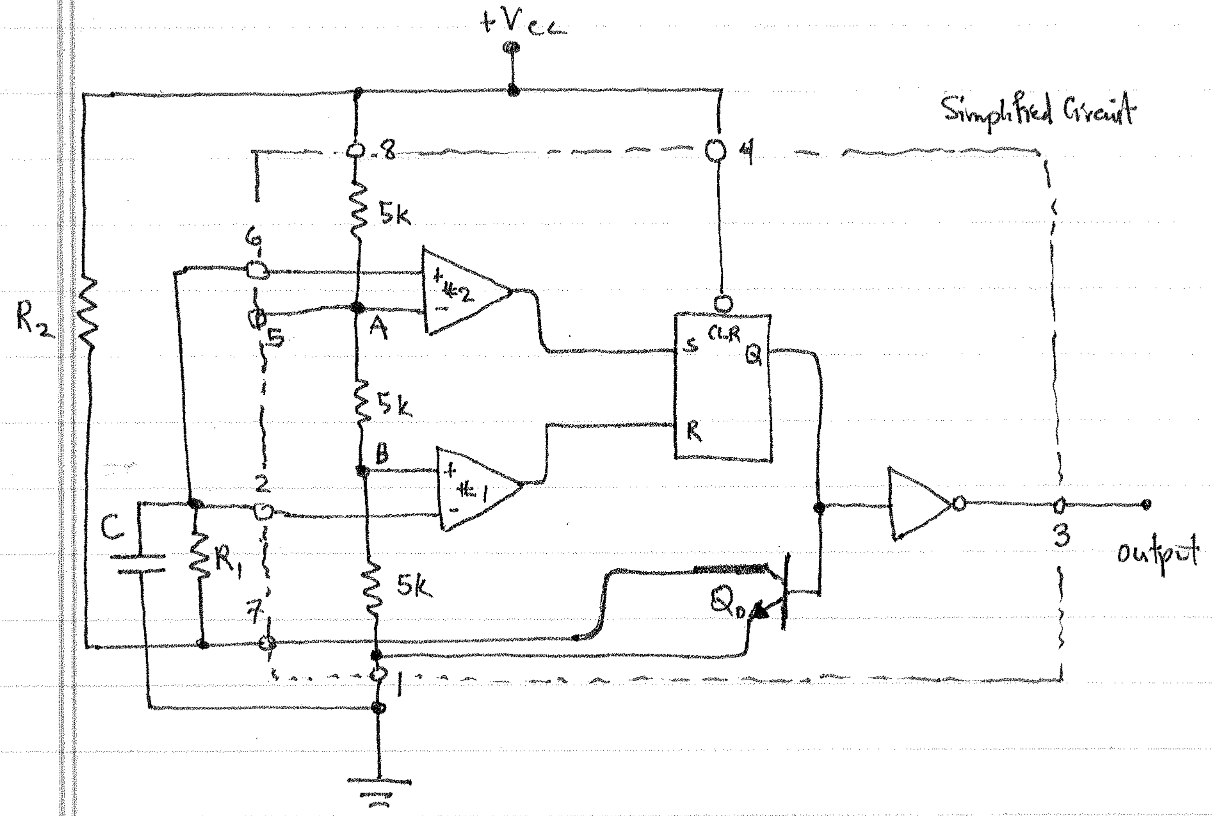
In time $\frac{T}{2}$, V_a decays from $+V_{cc}$ to $+\frac{V_{cc}}{2}$ ...ending up at $\frac{1}{2}$ its initial voltage (with respect to LO).

So $e^{-\frac{T}{2RC}} = \frac{1}{2}$

or $T = 2 \ln 2 \, RC$

$\boxed{T \simeq 1.4 RC}$

- Using the 555 Timer Chip to generate a digital clock:


+Vcc
Simplified Circuit
8
4
5k
6
5
A
+
-
#2
CLR
S
Q
R
5k
B
+
-
#1
2
C
R_1
R_2
7
5k
Q_D
1
3
output

This circuit has

1) Voltage divider
2) RC charging/discharging circuit
3) transistor used as a switch
4) Op-amps used as comparators
5) Logic gate ... inverter
6) Flip-flop

→ Nice summary of the course!

S	R	Q
0	0	memory
0	1	0
1	0	1
1	1	undesirable

Note that $V_A = \frac{2}{3} V_{cc}$ and $V_B = \frac{1}{3} V_{cc}$. These reference voltage levels are compared to the voltage on pins 2 and 6.... voltage across ~~on~~ the capacitor!

- Start with no charge on capacitor $V_2 = V_6 = 0$

  $V_6 < V_A$ ... comparator #2 is LO .... S = 0 on FF. } reset

  $V_2 < V_B$ ... comparator #1 is HI ... R = 1 on FF }

  Output of FF, Q = 0 ... V_{out} = 1/HI

  Transistor is OFF

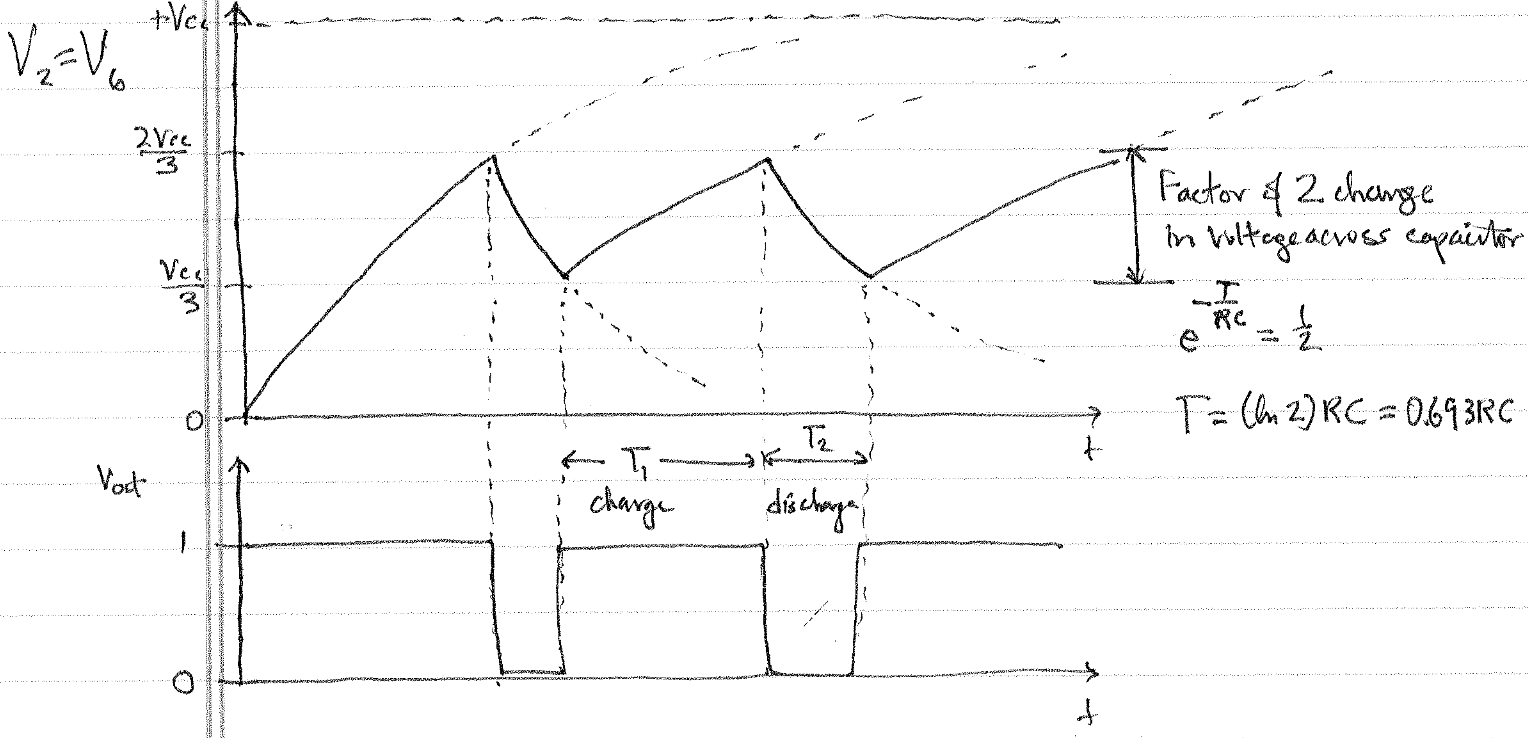
  Capacitor charges through $R_1 + R_2$.
- When $V_2 = V_6 = \frac{1}{3}V_{cc}$ then R = 0. FF is in memory state ... no change in output
- When $V_2 = V_6 = \frac{2}{3}V_{cc}$ then S = 1 / R = 0 FF is in set state

  Output of FF Q = 1 ... V_{out} = 0/LO

  Transistor is ON

  Capacitor discharges through R_1 ... S → 0 memory
- When $V_2 = V_6 = \frac{1}{3}V_{cc}$ ... R → 1 .. reset .. transistor off ... V_{out} = 1/HI

  capacitor charges


+Vcc
$V_2 = V_6$
$\frac{2V_{cc}}{3}$
$\frac{V_{cc}}{3}$
0
t
V_{out}
1
0
t
T_1
charge
T_2
discharge

Factor of 2 change in voltage across capacitor

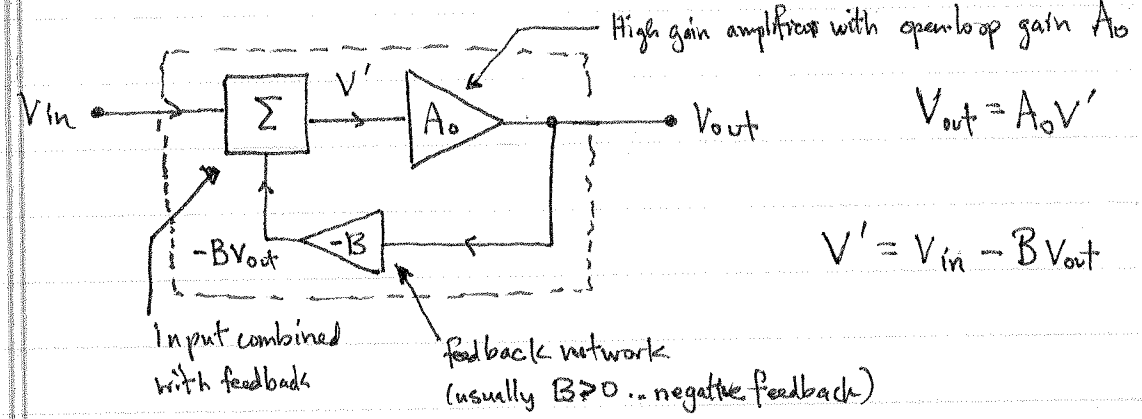
$$e^{-\frac{T}{RC}} = \frac{1}{2}$$

$$T = (\ln 2)RC = 0.693RC$$

$$T_1 = 0.69\,(R_1 + R_2)\,C$$
$$T_2 = 0.69\,R_1 C$$

- ## Positive Feedback and the Wien Bridge Oscillator

Block diagram for amplifier employing feedback


High gain amplifier with open-loop gain A_0
V_{in}
Σ
V'
A_0
V_{out}
$-B$
$-BV_{out}$
Input combined with feedback
feedback network
(usually B>0 .. negative feedback)

$$V_{out} = A_0 V' \quad (1)$$

$$V' = V_{in} - B V_{out} \quad (2)$$

Combine equations (1 + (2) $\quad V_{out} = A_0 V_{in} - A_0 B V_{out}$

or...
$$\boxed{V_{out} = \frac{A_0}{(1 + A_0 B)} V_{in}}$$

If $B>0$ ... negative feedback ... things are fine.

But, if $B<0$, then things go haywire when $|B| = \frac{1}{A_0}$

→ positive feedback!! even if Vin is not present

Since B can be frequency-dependent this condition might pertain

(a) at one (or a narrow range of) frequency (-ie s)

→ sinusoidal oscillations at that frequeny

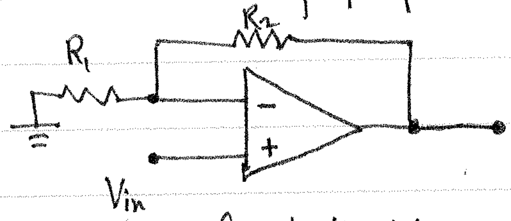
(b) over a wide range of frequencies

→ distorted or perhaps square-wave oscillations.

Example: The Wien Bridge Oscillator.

- Wien Bridge Oscillator

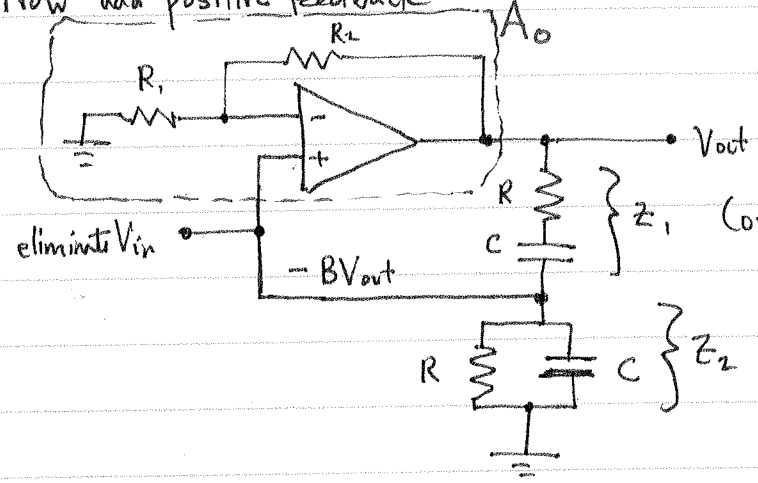
Start with a non-inverting opamp circuit



$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$

Consider this to be our high gain amp with $A_0 = \left(1 + \frac{R_2}{R_1}\right)$

Now add positive feedback



Complex voltage divider!

Same value for R and C in Z_1 and Z_2.

$-BV_{out} = \frac{Z_2}{Z_1 + Z_2} V_{out}$ or.. $B = \frac{-Z_2}{Z_1 + Z_2}$

or $B = \frac{-1}{\left(\frac{Z_1}{Z_2}\right) + 1}$

$Z_1 = R - \frac{j}{\omega C}$ $Z_2^{-1} = \frac{1}{R} + j\omega C$

So.. $\frac{Z_1}{Z_2} = \left(R - \frac{j}{\omega C}\right)\left(\frac{1}{R} + j\omega C\right) = 2 + j\left[\omega RC - \frac{1}{\omega RC}\right]$

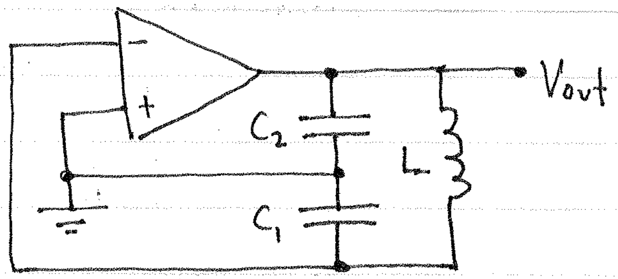
and $B = \frac{-1}{3 + j\left[\omega RC - \frac{1}{\omega RC}\right]}$ Will get instability/oscillation threshold when B is negative and real.. when $\omega RC - \frac{1}{\omega RC} = 0$

or… [illegible] $\omega = \frac{1}{RC}$ $\boxed{f = \frac{1}{2\pi RC}}$

AND then $B = -\frac{1}{3}$ … Also need $|A_0 B| = 1$ so… $A_0 = 3$

or… $\boxed{\frac{R_2}{R_1} = 2}$ at threshold

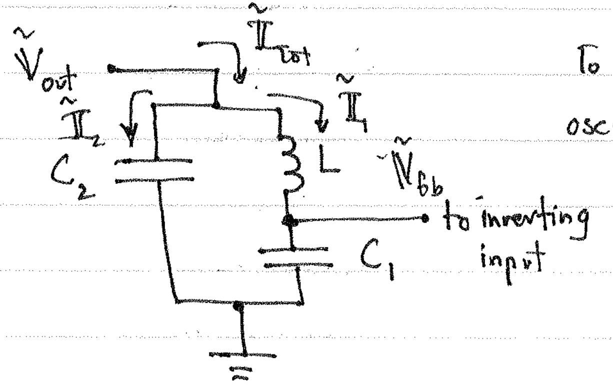
- Example of an LC oscillator using an Op Amp : Colpitts Oscillator



The feedback network is an LC "tank" circuit with a resonant frequency at $\omega_0 = \frac{1}{\sqrt{L C_{eq}}}$ C_{eq} is the equivalent capacitance of C_1 and C_2 in series.

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad \text{so...} \quad f_0 = \frac{1}{2\pi} \frac{1}{\sqrt{\frac{L C_1 C_2}{(C_1 + C_2)}}} = \boxed{\frac{1}{2\pi}\sqrt{\frac{C_1 + C_2}{L C_1 C_2}}}$$

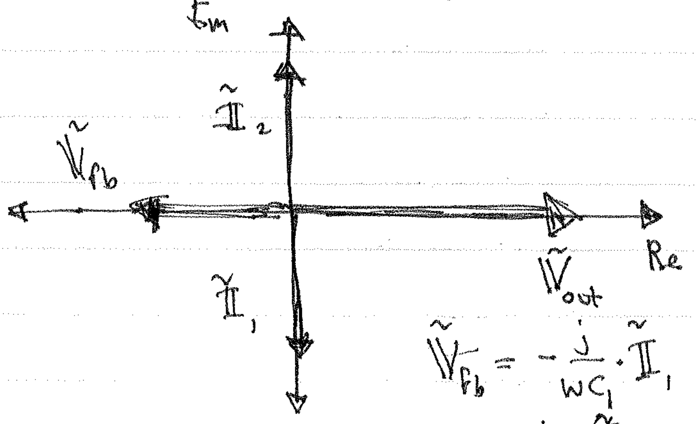
Redrawing feedback network



$\tilde{V}_{out}$
$\tilde{I}_{tot}$
$\tilde{I}_1$
$\tilde{I}_2$
C_2
L
$\tilde{V}_{Fb}$
to inverting input
C_1

To make feedback result in oscillations, the signal to inverting input of op-amp should be 180° out of phase with Vout... positive feedback.

Phasor diagram for currents/voltage at resonance.



Im
$\tilde{I}_2$
$\tilde{V}_{Fb}$
Re
$\tilde{V}_{out}$
$\tilde{I}_1$

$$\tilde{I}_2 = \frac{\tilde{V}_{out}}{Z_2} = j\omega C_2 \tilde{V}_{out}$$

$$\tilde{I}_1 = \frac{\tilde{V}_{out}}{Z_1} = \frac{-j\tilde{V}_{out}}{(\omega L - \frac{1}{\omega C_1})}$$

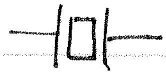
at resonance $\tilde{I}_{tot} = \tilde{I}_1 + \tilde{I}_2 = 0$

$(Q \rightarrow \infty)$

$\tilde{V}_{Fb} = -\frac{j}{\omega C_1} \cdot \tilde{I}_1$ 90° behind $\tilde{I}_1$

and $\tilde{I}_1$ is 90° behind $\tilde{V}_{out}$

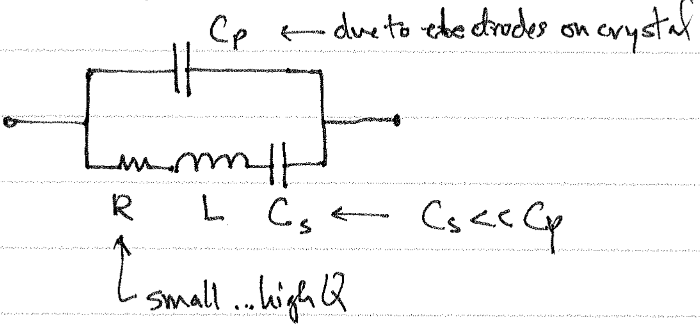
- Crystal Oscillators: Quartz crystals can have mechanical (vibrational) resonances with very high quality factors... $Q > 10^5$. The resonance can be coupled to an electric circuit because of the piezoelectric effect where the mechanical strain associated with the crystal's vibration results in voltage across the crystal.

Circuit symbol for crystal 

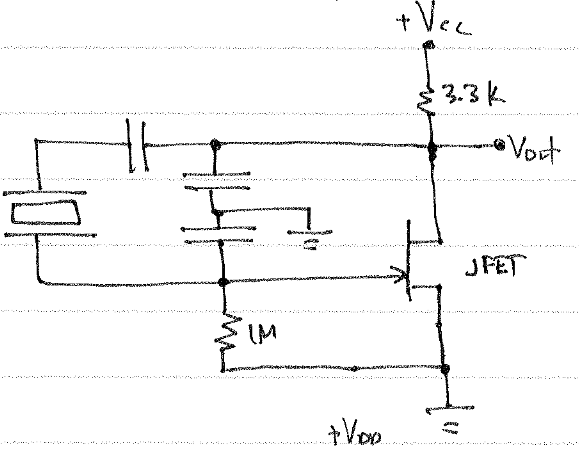
Electronic equivalent

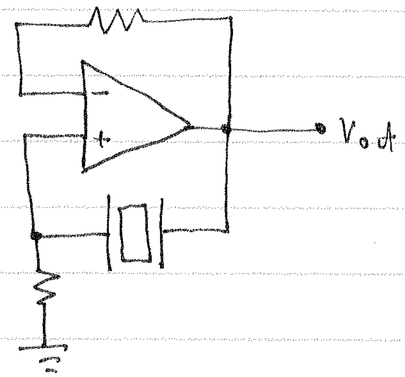
L ⟷ inertial component of oscillation

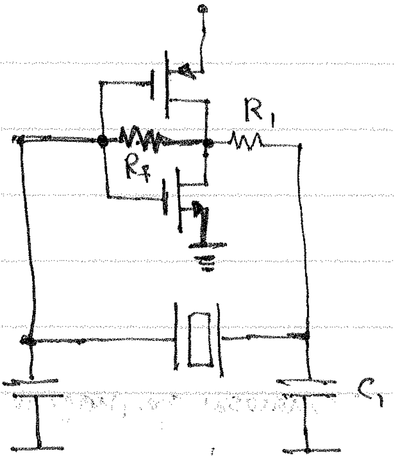
$\frac{1}{C_s}$ ⟷ elastic component of oscillation


C_p ← due to electrodes on crystal
R
L
C_s ← $C_s << C_p$
small ... high Q

Some oscillator circuits using crystal


+Vcc
3.3 k
Vout
JFET
1M



CMOS inverter-based crystal oscillator