Review for the Second Midterm

The second midterm exam will cover chapter 7 (the wave equation), chapter 8 (method of characteristics), chapter 9 (Green's functions), and section 4.3.3 (Duhamel's principle). Here is a list of topics to review for the midterm

- D'Alembert's method
- Solving the wave equation by Fourier series
- Solving a linear first order PDE by the method of characteristics
- Computing a Green's function by converting a known solution to the Green's function form
- The delta function and the sifting property
- The Heaviside function and its relation to the delta function
- Duhamel's principle
- Computing a Green's function by solving a problem with a delta function as the forcing function

Some Representative Sample Questions

1. The wave equation for a vibrating string

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t)$$

assumes that vibrating string is not subject to friction. The wave equation for a vibrating string in the presence of friction takes the form

$$\frac{\partial^2 u(x,t)}{\partial t^2} + \kappa \frac{\partial u(x,t)}{\partial t} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = f(x,t)$$

Explain how you would solve this equation. Suppose that for this problem we have that $f(x,t) = 0$, $u(0,t) = u(l,t) = 0$, $u(x,0) = x\,(l - x)$, $\frac{\partial}{\partial t} u(x,0) = 0$.

2. The string on a piano is 1 meter in length and c is 20 meters per second. The ends of the string are fixed and the string is at rest at time t=0. A pianist strikes a key so as to impart a velocity $g(x)$ where

$$g(x) = \begin{cases} 10 & 0.6 < x < 0.7 \\ 0 & 0 \le x < 0.6,\ 0.7 < x \le 1 \end{cases}$$

What is the displacement $u(x,t)$ for the string at times $t \ge 0$?

3. Use the method of characteristics to solve

$$\frac{\partial u(x,y)}{\partial x} - \frac{\partial u(x,y)}{\partial y} + u(x,y) = 0$$

$$u(x,0) = \sin x$$

4. Compute a Green's function for the BVP

$$u''(x) + u(x) = f(x)$$

$$u(0) = u(l) = 0$$

by solving the problem

$$u''(x) + u(x) = \delta(x-y)$$

$$u(0) = u(l) = 0$$

5. Develop a Green's function for the wave equation with friction term shown in problem 1.