Physics 225 COMPUTATIONAL MECHANICS Winter Term, 2006

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REQUIRED TEXTS: The following three items are required for this course and will be distributed at the first class. Two or three weeks into the term, after the initial ups and downs of registration have subsided, I will instruct the Business Office to charge your student account for these materials. I will, of course, have also to add the 5% Wisconsin sales tax. The total charge posted to each of your accounts will be $52.50.

- *Computation and Problem Solving in Undergraduate Physics* (*CPSUP*) by David M. Cook; $30.00. You will have either (1) the edition dated 18 December 2005 (494 + xxiv pages) or (2) the edition dated 19 December 2004 (492 + xxiv pages) and a supplement containing an updated version of Section A.9 and an updated version of Appendix B. Both editions contain all components of *CPSUP* that are pertinent to Lawrence. We will, however, make use of only a portion of the book. I am providing all of it on the confident assumption that the rest will be valuable to you as you continue your studies of physics at Lawrence. I believe this book is one you will want to make use of throughout your time at Lawrence (and perhaps beyond). You will first need *CPSUP* when you start Assignment 2 on Friday, 13 January, but I have the materials to deliver to you at the start of the term.

- *Notes for Computational Mechanics* (*NCM*) (Edition dated 8 December 2005; 188 + xiv pages), David M. Cook; $13.00. This locally produced manuscript is the primary text for the mechanics taught in this course. The alternative would be a formally published text, which would cover much more mechanics than we will cover and would be much more expensive. For your reference, some of the possible texts that *might* have been chosen are made available on reserve in the library.

- *Local Guide* (*LG*) to accompany *CPSUP* (Edition dated 2 January 2006; 70 pages); $7.00. This guide has suffered substantial revision from last year's edition so as to incorporate the new Hewlett-Packard workstations with which the Computational Physics Laboratory (CPL) is now equipped. Because of those edits, the guide did not go to press until 2 January 2006. If I don't have copies of the printed and bound guide for you at the first class, I will have xeroxed copies of the portions that you need in the first week. If it is not available at the first class, the full guide will be available shortly thereafter.

Versions of these materials from past years will *not* do; significant edits have been made since previous versions were printed.

RECOMMENDED SUPPLEMENTARY TEXT: R. Shankar, *Basic Training in Mathematics: A Fitness Program for Science Students* (Plenum Press, New York, 1995; ISBN 0-306-45036-4 (paper)). Taking a physicist's perspective, this book reviews a lot of mathematics, most of which you already know at some level. The Department has agreed that the book is a worthwhile addition to your personal library. Assignments in Physics 225 will occasionally direct your attention to portions of the book as supplementary reading. The book will also be referred to in subsequent physics courses. I think it would be worthwhile for you pick the book up and read a few pages every once in awhile, simply to help you develop increased familiarity with the mathematics pertinent to physics. Treat it as a source of bedtime entertainment. There is (supposed to be) one copy of the book in the CPL. There is a second copy on the Physics 225 reserve shelf at the main library. If, after looking the book over, you agree you should have a personal copy, feel free to order it however you normally acquire your books.

SCHEDULE: The schedule for this course is included with this fact sheet. Class will meet three times a week. Occasionally, when you are first learning some computer program, a class will be cancelled so as to compensate you for the time you spend at the keyboard doing an extended exercise on your own. (We simply don't have enough computer hardware to conduct in-class workshops.)

OFFICE HOURS: Though they may require adjustment once one or two further scheduling items settle for the term, at the moment I set the following office hours:

7:00–9:00 PM most Sundays
1:15–2:45 PM most Mondays
9:00–10:30 AM most Thursdays
other times by appointment

CONSULTING SCHEDULE: Very soon after the start of the term, Erik Garbacik will be on duty in the CPL to provide assistance with the computer portions of Physics 225. The first few times when he will be available are included in the schedule later in this fact sheet. Times later in the term will be announced in class and posted on the bulletin board next to the door into the CPL.

CONFERENCE SESSIONS: Five conference sessions are scheduled at intervals through the term. These sessions provide opportunity for detailed discussion of the current material. In addition, in each session, two or three students will be asked to present solutions to designated problems. No new material will be presented in these sessions. So that everyone will have at least two opportunities to make presentations, the class may for these conferences be divided into two sections, one of which will meet at the regular class hour and the other of which will meet at a time to be arranged.

ASSIGNMENTS: Seven problem assignments, all of which are described later in this fact sheet, are distributed during the term. Each assignment identifies readings in *NCM*, *CPSUP*, and *LG*, occasionally suggests readings in alternative texts (including Shankar), and—most important—designates several problems that you should solve as a test that you understand the material covered by the assignment. Problems are divided into two groups. You should make a serious effort to solve *all* of the problems in Group 1 and ask about any that you are unable to solve. Think of problems in Group 2 as interesting diversions to be attacked only after you have mastered the remainder of the assignment. The problems in Group 1 are to be written up *carefully*, *thoroughly*, and *neatly*. So that I don't have to spend time writing *extensive* comments on your solutions and so that you can see what may be alternative approaches to the problems, a day or so after each assignment is due I will place solutions for the problems in Group 1 (and perhaps some of the problems in Group 2) in a notebook that will live in Y-104/CPL. *In the interests of your keeping abreast of the course and of my efficient grading, I ask that you turn in at each due date whatever of the assignment you have completed; do not withhold an entire assignment solely because you have not completed some of the problems.*

Please recognize from the outset that I believe a solution to a problem is FAR more than a mere answer, that the method by which you solve a problem is more important than *the end result. All submitted solutions must be* complete*, i.e., they must contain not only the answer, not only a succession of equations, but also—and most important—enough narrative to make clear how you thought your way carefully and logically from first principles. It is your job in your submitted work to convince me that you have identified the applicable principles, recognized clearly what those principles are, and worked your way step-by-step from that starting point to the solution. Pay attention not only to the substance of your solution but also to its exposition. Solutions that look like your first draft on a sheet of scrap paper are not acceptable.*

Further, note that, when you include computer-produced graphs in a solution, your solution will be incomplete unless the accompanying narrative describes how you produced the graphs and indicates the specific statements you submitted to whatever program you used.

As a test of adequacy, ask yourself whether you would accept and understand your completed solution were I to present it to the class by saying nothing and only writing on the board exactly what you have written as your solution.

Finally with regard to problems, I suggest that you read *all* of the problems in the text. Though I will assign only a subset of those problems, taking the time to read all of them will help give you a fuller idea of the wide variety of problem that can be successfully addressed with the conceptual framework we will be developing.

LIBRARY RESERVE: Several books are on reserve for your use in the main library. Copies of *some* of these materials are also available in the library in the CPL. In addition, vendor manuals for the software packages we will be using are on the shelves in the CPL. (See the bibliographies at the ends of chapters in *CPSUP*.) You may wish to read portions of these materials to supplement your study of *LG*, *NCM*, and *CPSUP*. Possibly useful supplementary materials include:

- Barger and Olsson, *Classical Mechanics: A Modern Perspective*
- Marion, *Classical Dynamics*
- Slater and Frank, *Mechanics*
- Goldstein, *Classical Mechanics*
- Feynman, *Lectures in Physics, Volumes I and II*
- Symon, *Mechanics*
- Shankar, *Basic Training in Mathematics*

Fuller citations for all of these materials will be found in the bibliography on pages ix and x in *NCM*.

COMPUTER: Recognizing the growing importance of skills in using assorted computational resources both to the upper-level physics curriculum at Lawrence and to the contemporary practice of physics, engineering, and other sciences, the Department of Physics introduced Physics 225 (*Computational Mechanics*) in the winter of 2003. Physics 225 replaces the former Physics 21/210 (*Mechanics*). The primary objective of this change is to provide a firmer and more universal introduction to computational approaches to problems in physics than was possible with the former elective course *Computational Tools in Physics* (Physics 27/270, 28/272, and 29/274). Coverage of some of the mechanics (general rigid body motion, Lagrangian and Hamiltonian mechanics, collisions and scattering, ...) previously treated in Physics 21/210 will of necessity be curtailed or deleted altogether to make room for the addition of topics in graphical visualization, numerical solution of ordinary differential equations, numerical evaluation of integrals, and publishing with a scientifically sophisticated text processing system (LaTeX).[1] On several occasions during the term, class sessions will provide orientation and assignments will expect you to work through computer-based tutorials and make use of computer-based approaches to assigned problems. As you learn about the capabilities of IDL, MAPLE, and LaTeX, please feel free to use these resources not only to address exercises that direct you specifically to the computer but also whenever else it seems appropriate to you to exploit our computational resources. The more you practice, the more fluent and comfortable you will become.

Note particularly the exercise (SP1.1) on the *first* assignment that asks you to familiarize yourself with the workstations, set up your account, and explore one or two simple applications. There is nothing to be handed in when you finish SP1.1. Nonetheless, please make it a point to do that exercise as soon as possible. The *second* assignment will expect you to have developed the beginning skills that are the aim of SP1.1.

EXAMINATIONS: There will be two hour examinations, and a final examination in this course. All examinations will have an in-class, closed-book portion. In addition, each examination—including the final examination—*may* have a take-home portion that will be open-book but closed classmate and will contain one or more computer-based exercises to be done out of class time.

GRADING: Your grade in this course will be determined largely by your performance on the examinations and by your diligence and success in working the assigned problems. In particular, a final grade of A will be recorded only for those students who do exceptionally well on all three examinations and who *in addition* complete a substantial fraction (at least 90%) of the assigned problems successfully. Failure to complete some reasonable fraction (say 70-80%) of the assigned problems may depress whatever grade is earned on the examinations alone by as much as a full letter grade.

HONOR SYSTEM: Each student is expected to present only her or his own work on the hour examinations and the final examination. In contrast, students are encouraged to work together on the assignments. Each student is expected to write up her or his own assignments, but working together to solve the problems can be a valuable learning aid. I establish only two ground rules: (1) Working together will be most effective if all individuals contribute more or less equally to the group effort; you should be wary if you are always on the receiving end in such effort, for ultimately you will be expected to perform on your own. At the very least, once you have finished solving an exercise in a group, make sure that you could solve a similar exercise on your own. (2) Where substantial help has been received through conversation with another, I ask that you follow common scientific courtesy and acknowledge that help briefly in your submitted work.

[1] The omitted topics in mechanics are now included in the junior/senior elective course Physics 410, *Advanced Mechanics*, which is offered in alternate years.

Physics 225 SCHEDULE Winter Term, 2006

Abbreviations used in this schedule are C for conference session, H for hour examinations, N for no class, L for lecture, P for problems due, *NCM* for *Notes for Computational Mechanics*, *CPSUP* for *Computation and Problem Solving in Undergraduate Physics*, and *LG* for the *Local Guide*. Unless otherwise noted, assignments are due at the start of class on the indicated days.

04 Jan	We	L: Kinematics, Dynamics, and Newton's Laws (*NCM* pp 101–112, 201–214)
05 Jan	Th	**Erik Garbacik available in the CPL** (8:00-10:00 PM)
06 Jan	Fr	L: (previous topic continued) (*NCM* 214–230)
08 Jan	Su	**Erik Garbacik available in the CPL** (8:00-10:00 PM)
09 Jan	Mo	L: Forces and Simple Applications without ODEs (*NCM* pp 301-324)
11 Jan	We	C: Conference on Assignment 1
13 Jan	Fr	P: ASSIGNMENT 1 DUE N: NO CLASS; Work with IDL and TGIF in the CPL (*CPSUP* Chapter 2, Sections 1–12, and Appendix B)

****** 13–15 Jan Fr–Su DEPARTMENTAL BJÖRKLUNDEN RETREAT ******

15 Jan	Su	**Erik Garbacik available in the CPL** (8:00-10:00 PM)
16 Jan	Mo	N: NO CLASS — MARTIN LUTHER KING DAY
18 Jan	We	L: Equations of Motion (*NCM* pp 401–414; *CPSUP* Sections 9.1.1, 9.1.4, 9.1.6, and 9.1.7)
19 Jan	Th	**Erik Garbacik available in the CPL** (8:00-10:00 PM)
20 Jan	Fr	P: ASSIGNMENT 2 DUE L: Constant, Time-Dependent, and 1D Position-Dependent Forces (*NCM* pp 415–425)
23 Jan	Mo	L: Potential Energy and 3D Position-Dependent Forces (*NCM* pp 425–440)
25 Jan	We	L: Velocity-Dependent Forces and Damped SHM (*NCM* pp 440–449)
27 Jan	Fr	L: Driven SHM and Resonance, Coupled Oscillators (*NCM* pp 448–464)
30 Jan	Mo	C: Conference on Assignment 3
01 Feb	We	P: ASSIGNMENT 3 DUE H: HOUR EXAMINATION #1 (covering Assignments 1, 2, 3)
03 Feb	Fr	L: LaTeX for Scientific Publication (*CPSUP* Appendix A)

06 Feb	Mo	L: Motion in a Plane, Central Forces, Effective Potentials, and the Orbital Equation (*NCM* pp 501-515)
08 Feb	We	L: Planets, Satellites, Comets (*NCM* pp 516–536) (On this Wednesday, I must be on board a plane that leaves Appleton at 10:15 AM so I can be part of an outside review team at Moravian College, Bethlehem, PA. A class running until 9:40 AM renders that objective unachievable, so we must find an accommodation. I suggest two possibilities: (1) Class meets Tuesday evening or (2) class meets from 8:00–9:00 AM on Wednesday morning. We will discuss these and any other options you are moved to suggest early in week 2 of the term.)
10 Feb	Fr	NO CLASS; MIDTERM READING PERIOD
13 Feb	Mo	C: On Assignment 4
15 Feb	We	P: ASSIGNMENT 4 DUE AT NOON N: NO CLASS; Work with MAPLE in the CPL (*CPSUP* Chapter 5, Sections 1–8, 11, and 12)
17 Feb	Fr	L: Using MAPLE to Solve ODEs (*CPSUP* Section 9.4)
20 Feb	Mo	ASSIGNMENT 5 DUE L: Algorithms for Solving ODEs Numerically (*CPSUP* Section 9.6)
22 Feb	We	N: NO CLASS; Work with IDL in the CPL (*CPSUP* Section 9.7)
24 Feb	Fr	L: Using IDL to Solve ODEs Numerically (*CPSUP* Section 9.7)

****** 24–26 Feb Fr–Su LPW RECRUITING WEEKEND ******

27 Feb	Mo	C: Conference on Assignment 6
01 Mar	We	P: ASSIGNMENT 6 DUE H: HOUR EXAMINATION #2 (covering Assignments 4, 5, 6)
03 Mar	Fr	L: Using MAPLE to Evaluate Integrals Symbolically (*CPSUP* Sections 11.1 and 11.3)
06 Mar	Mo	L: Algorithms for Evaluating Integrals Numerically (*CPSUP* Section 11.5)
08 Mar	We	L: Using IDL to Evaluate Integrals Numerically (*CPSUP* Section 11.6)
10 Mar	Fr	C: Conference on Assignment 7
11 Mar	Sa	P: ASSIGNMENT 7 DUE
14 Mar	Tu	(7:00–9:00 PM) Special Office Hours
15 Mar	We	8:30 AM Final Examination

ASSIGNMENT 1 (due Friday, 13 January 2006)

Reading:	*NCM* Sections 1.1, 1.2, and 1.3, Chapter 2, and Chapter 3
	LG, especially Sections 1, 2, 3, 4, 5, and 18.
Suggested:	Shankar, Chapter 1, pp 1–29
	Shankar, Chapter 7, pp 149–159
Problems 1:[2]	SP1.1, SP1.2,
	P2.4, P2.15, P2.19, P2.21, P2.29, P2.34,
	P3.8 [omit part (c)], P3.13, P3.15, P3.19
Problems 2:	P2.6, P2.17, P2.20, P2.23, P3.3, P3.5, P3.17

SP1.1. Work your way through the "whirlwind tutorial" contained in Section 2 of the *Local Guide*. This tutorial helps you determine your user name and initial password, leads you through the login process, helps you change your password, and then guides you through a few exercises designed to familiarize you with the rudimentary capabilities of software available in the CPL. There is nothing to *hand in* for this "problem", but it must be done by the due date for this assignment, since subsequent assignments will assume that you are already familiar with the contents of this tutorial. Note also the schedule when a consultant will be in the CPL to offer assistance; the first two or three times you log in, it is useful to have a more experienced individual nearby.

SP1.2. More to refresh your understanding and set notation than to learn the material for the first time, read Sections 1.1, 1.2, and 1.3 in *NCM*. I will quickly review especially the notation of coordinate systems in class, but I do not intend to dwell on this part of the first assignment. Pay particular attention to Eqs. (1.2), (1.3), (1.9, (1.12), (1.13)–(1.18), and (1.20), Fig. 1.5, and Table 1.1. As with SP1.1, there is nothing to hand in related to this material. If, however, you want to test your understanding, you might attempt a selection of problems chosen from P1.4, P1.5, P1.7, P1.15, P1.17, and P1.20.

ASSIGNMENT 2 (due Friday, 20 January 2006)

Reading:	*CPSUP* Chapter 2, especially Sections 1–3, 5, 7–12
	CPSUP Appendix B

> *N. B.*: The example statements to the computer included in the text are intended to be executed. Don't fail to enter those statements, observe their effect, and perhaps explore the behavior of slightly modified statements before addressing the problems to be handed in.

Problems 1:[3]	2.8, 2.14, 2.18, 2.24, 2.30, B.2
Problems 2:	2.12, 2.21, 2.25, P3.8 [part(c)]

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[2] Problems whose number is preceded with the characters SP will be presented on the assignment sheet itself; SP1.1, for example, is just below the listing of the problems. Problems whose number is preceded with the character P are in *NCM*, either at the end of the appropriate section or at the end of the appropriate chapter.

[3] Problems whose number is preceded by no special character are in the indicated chapter of *CPSUP*; The problem numbered 2.8, for example, is at the end of Chapter 2 in *CPSUP*; Problem B.2 is at the end of Appendix B in *CPSUP*.

ASSIGNMENT 3 (Wednesday, 1 February 2006)

N. B.: This is a particularly long assignment. Do not postpone starting it. Problems occur at the end of the sections containing the necessary background, so you can tell easily from your readings and from the content of class sessions when you should be able to do each problem. Do *not* wait until two days before the assignment is due to start the problems. Such behavior will inevitably lead to frustration, a couple of long evenings, and an incomplete assignment.

Reading:	*NCM* Chapter 4 and, for review, *NCM* Section 1.4
	CPSUP Sections 9.1.1, 9.1.4, 9.1.6, and 9.1.7
Suggested:	Shankar, Chapter 7, pp 159–162, 167–181
	Shankar, Chapter 3, pp 51–54; Chapter 10, pp 305–312
	Shankar, Chapter 8, pp 205–219; Chapter 9, pp 255–266
Problems 1:	P4.13, P4.27, P4.28, P4.29, P4.30, P4.41, P4.44, P4.50,
	P4.51, P4.60, P4.63, P4.70
Problems 2:	P4.26, P4.38, P4.39, P4.40, P4.42, P4.43, P4.49, P4.56,
	P4.64, P4.65, P4.71, P1.31

ASSIGNMENT 4 (due Wednesday 15 February 2006 at noon)

Reading:	*CPSUP* Appendix A
	NCM Chapter 5
Problems 1:	A.4*,†, P5.1, P5.5†, P5.6, P5.12, P5.13,
	P5.17, P5.31†, P5.32, P5.34, P5.38
Problems 2:	P5.3, P5.15, P5.20, P5.22, P5.23, P5.28

* Remember that, at Lawrence, the symbol `$HEAD` used in *CPSUP* must be translated to `/usr/apps/CPSUP`.

† Problems marked with this symbol—a dagger †—are to be written up using LaTeX. Feel free to prepare other problems with this tool as well, but I will mandate that only the three here marked *must* be done this way.

ASSIGNMENT 5 (due Monday, 20 February 2006)

Reading:	*CPSUP* Sections 5.1–5.8 (especially 5.8.9), 5.11, 5.12
	CPSUP Section 9.4
Problems 1:	5.5, 5.12, SP5.1, SP5.2, 9.2
Problems 2:	5.7, 5.16, 9.7

SP5.1. Using MAPLE, obtain graphs of the potential in P4.27 and the associated force. Note that the expression for the potential is simpler to plot if you plot not $U(x)$ versus x but rather $V(x)/V_0$ versus $\overline{x} = x/a$, i.e., rewrite the function in the form

$$U(x) = -\frac{U_0 a^2(a^2+x^2)}{8a^4+x^4} \qquad \longrightarrow \qquad \frac{U(x)}{U_0} = -\frac{1+\overline{x}^2}{8+\overline{x}^4}$$

You should be able to find a similar dimensionless version of the expression giving the force. Be sure your submitted solution explains fully how you used MAPLE to generate the desired graphs. (*Note*: There are numerous ways to obtain hard copy from MAPLE. Probably the easiest involves working in the GUI interface (started with the command `xmaple`) because, in that interface, you can

- Enter commands at the MAPLE prompt.

- Hide all your false starts by removing commands that don't work. (Select the erroneous paragraph by clicking ML on the opening square bracket that delineates it and then select 'Delete Paragraph' from the EDIT menu.)
- Enter textual descriptions as described in *CPSUP* Section 5.14.
- Display graphs inline rather than in a separate window by executing the command `plotsetup(inline)` to MAPLE.
- Print the entire contents of the window by selecting 'Print' from the FILE menu. (In the *Printer Setup* window that comes up, select 'Print Command' and make sure the command in the text entry box is `lp`, select 8.5"×11" paper size, and select 'Portrait' for the page layout before clicking ML on the 'Print' button.)

The contents of the MAPLE window can also be exported to a file in several formats, some of which can be processed with LaTeX. Those issues are discussed in Section 5.13, which you should feel free to explore even though it is not officially assigned.

SP5.2. Using MAPLE, find complete (symbolic) solutions to each of the following problems, and use MAPLE also to verify that the solutions you obtain actually *do* satisfy the original ODE and initial conditions.

a. $\dfrac{d^2x}{dt^2} = a$, $x(0) = x_0$, $v(0) = v_0$, where a is constant, i.e., find position as a function of time for a particle moving under the action of a constant force and launched with arbitrary initial conditions.

b. $m\,\dfrac{d^2x}{dt^2} = -eE_0\cos(\omega t + \theta)$, $x(0) = x_0$, $v(0) = v_0$, i.e., find position as a function of time for a charged particle moving under the action of a sinusoidal force and launched with arbitrary initial conditions.

c. $m\,\dfrac{d^2x}{dt^2} = -mg + b\left(\dfrac{dx}{dt}\right)^2$, $x(0) = 0$, $v(0) = 0$, i.e., find position as a function of time for a particle released from rest at the origin and allowed to fall freely under the action of gravity and a viscous retarding force proportional to the *square* of the speed.

d. the differential equations for $x(t)$ and $z(t)$ in P4.50.

Be sure your submitted solution explains fully how you used MAPLE to generate the desired solutions. (*Note*: See the note in Problem SP5.1.)

ASSIGNMENT 6 (due Wednesday 1 March 2006)

Reading:	*CPSUP* Sections 9.6 and 9.7
Suggested:	Feynman, Volume 1, Lecture 9
Problems 1:[4]	SP6.1†, SP6.3†, 9.18†, any two—your choice—of (9.17†, 9.19†, 9.23†)
Problems 2:	SP6.4, SP6.5, the other of (9.17, 9.19, 9.23), 9.27

† Solutions to these exercises are to be prepared using LaTeX.

SP6.1 (See also P5.12) A particle of mass m moves under the action of a central force whose potential is

$$U(\rho) = K\rho^4, \qquad K > 0$$

Find the equations of motion for a planet moving under the influence of this force, cast them in suitable dimensionless form, and use numerical methods to explore both circular orbits (as verification of your numerical procedures) and orbits having other initial conditions. Try approaching the problem

[4] Problems whose number is preceded by no special character are in the indicated chapter of *CPSUP*; the problem numbered 9.17, for example, is at the end of Chapter 9 in *CPSUP*.

both in Cartesian coordinates and in polar coordinates. Write several paragraphs describing and presenting evidence for your discoveries.

SP6.3. In P4.27, we found that the potential

$$U(x) = -U_0 \frac{a^2(a^2+x^2)}{8a^4+x^4} \qquad \text{or} \qquad \frac{U(x)}{U_0} = -\frac{1+\overline{x}^2}{8+\overline{x}^4}$$

where $\overline{x} = x/a$ exhibited a variety of motions (bound motion in either the left or the right well when $-\frac{1}{4} \le E/V_0 \le -\frac{1}{8}$; bound motion over the whole well when $-\frac{1}{8} \le E/V_0 \le 0$; unbound motion when $E/V_0 > 0$). Find the equation of motion for this system, cast it in a suitable dimensionless form, and then explore each of the different types of motion possible. Be sure to look both at graphs of x and $v = dx/dt$ versus t *and*—especially for the periodic motions—at the phase plane plots, v versus x—all in suitable dimensionless units. It might be interesting to examine period as a function of energy for this oscillator. Write several paragraphs describing and presenting evidence for your discoveries. *Hint*: Perhaps a suitable dimensionless time would be $\overline{t} = \omega_0 t$, where ω_0 is the frequency of small amplitude oscillations when the particle is confined to one side of the well.

SP6.4. A 1000 kg mass is dropped on a platform from a height of 10 m (acquiring a velocity of 14.0 m/s as it hits the platform). The spring is characterized by a constant $k = 49000$ N/m and the mass strikes the platform a distance $x_0 = 0.2$ m above the desired equilibrium point. The undamped frequency of oscillation is $\omega_0 = 7.0$ rad/s and the ratio $\gamma = b/2m$ for *critical* damping is also 7.0 rad/s. The equation of motion is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

to be solved subject to the requirements $x(0) = x_0 = 0.2$ m and $v(0) = -14.0$ m/s. (a) Show that this problem can be recast in the dimensionless form

$$\frac{d^2\overline{x}}{d\overline{t}^2} + 2q\frac{d\overline{x}}{d\overline{t}} + \overline{x} = 0 \qquad ; \qquad \overline{x}_{\overline{t}=0} = 1 \qquad ; \qquad \left.\frac{d\overline{x}}{d\overline{t}}\right|_{\overline{t}=0} = -10$$

where $\overline{x} = x/x_0$, $\overline{t} = \omega_0 t$, and $q = b/2\omega_0 m = \gamma/\omega_0$. (Note that $q = 1$ for the case of *critical* damping.) (b) Explore the solution to this problem numerically for several values of q, including values both larger and smaller than 1. In particular, note the overshoot when $q = 1$ and search for the value of q at which overshoot first disappears. What is the value of b at that point?

SP6.5. One type of child's swing is hung with elastic ropes. Suppose the ropes are long enough so that the child/swing can be represented by a point mass at the end of a spring, assume the spring obeys Hooke's law and has an unstretched length a, and let the motion of the mass be confined to a single vertical plane. Show that, in the coordinate system illustrated, the equations of motion are

$$m\frac{d^2x}{dt^2} = -kx + \frac{kax}{\sqrt{x^2+y^2}} \qquad ; \qquad m\frac{d^2y}{dt^2} = mg - ky + \frac{kay}{\sqrt{x^2+y^2}}$$

Then, introducing $\omega_0^2 = k/m$, $\overline{t} = \omega_0 t$, $\overline{x} = x/a$, and $\overline{y} = y/a$, cast the equations in dimensionless form. After the suggested rescalings, only one parameter—$g/a\omega_0^2$—remains [and *this* parameter is the (square of the) ratio of the frequency $\sqrt{g/a}$ of a simple pendulum of length a to the frequency $\sqrt{k/m}$ of a mass m bobbing on a spring of stiffness k]. Explore the motions for several values of this one parameter, including values larger than, equal to, and smaller than 1. Write several paragraphs describing and presenting evidence for your discoveries.

SP6.6. Use IDL to study the motion of a mass sliding on a horizontal surface under the action of a force from a spring and the force of dry friction. The equation of motion is

$$m\frac{d^2x}{dt^2} = -kx - \mu\, mg\, \mathrm{sgn}\left(\frac{dx}{dt}\right)$$

where $\mathrm{sgn}(z)$ is the signum function, $+1$ when $z > 0$, -1 when $z < 0$, and 0 when $z = 0$. Be particularly careful to lay out a criterion for determining when the motion has stopped. Write several paragraphs describing and presenting evidence for your discoveries.

SP6.7. Consider a damped pendulum but add a sinusoidal driving force. In dimensionless form, the equation of motion is

$$\frac{d^2\theta}{dt^2} + \Gamma\frac{d\theta}{dt} + \sin\theta = A\cos(\Omega t)$$

where the three parameters are Γ, which is a dimensionless measure of the damping; A, which is a dimensionless measure of the amplitude of the driving force, and Ω, which is the ratio of the frequency of the driving force to the small amplitude natural frequency of the undamped pendulum. This equation is conveniently recast in terms of the variables θ, $\omega = d\theta/dt$, and $\phi = \Omega t$, and we find that

$$\frac{d\theta}{dt} = \omega \qquad ; \qquad \frac{d\omega}{dt} = -\Gamma\omega - \sin\theta + A\cos\phi \qquad ; \qquad \frac{d\phi}{dt} = \Omega$$

This system exhibits an enormous range of behaviors, depending on the relationships among the three parameters. You should look particularly at the way the behavior changes with change in the amplitude A of the driving force. Thus, at least at the start—you may explore other values later if you wish—take $\Gamma = 0.5$ and $\Omega = \frac{2}{3}$. Study the *steady state*—i.e., wait long enough for any initial transients in the motion to have died out before plotting graphs—behavior of this system for various driving amplitudes, including $A = 0.5$, 1.07, 1.35, 1.45, 1.5. Look particularly at the phase plane plots ω versus θ. Write several paragraphs describing and presenting evidence for your discoveries.

ASSIGNMENT 7 (due Saturday 11 March 2006 at noon)

Reading:	*CPSUP* Chapter 11, Sections 1, 3, 5, and 6
Problems 1:[5]	11.4, 11.7, 11.10, 11.11,
	11.14, 11.18†, 11.24†, 11.27†, 11.30†
Problems 2:	11.2, 11.5, 11.6, 11.22, 11.25, 11.29

† Solutions to these exercises are to be prepared using LaTeX. Feel free to use LaTeX for other problems as well. Alternatively, for those problems that involve MAPLE, using the MAPLE notebook interface as an alternative to LaTeX or to handwriting the solutions would be fine.

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[5] Problems whose number is preceded by no special character are in the indicated chapter of *CPSUP*; the problem numbered 11.14, for example, is at the end of Chapter 11 in *CPSUP*.